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SIZE EFFECTS, PROCESS ZONE AND TENSION SOFTENING BEHAVIOR IN FRACTURE OF GEOMATERIALS

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Abstract—In this paper we review experimental and field observations of the process zone, K_{Ic} size dependence in concrete and rock and suggest that a valid K_{Ic} test for such materials may require impractically large specimen sizes. Direct application of linear elastic fracture mechanics (LEFM) to toughness testing of quasi-brittle geomaterials has not been consistently successful. An alternative material property—the tension softening curve—is better suited to characterize crack formation and growth in geomaterials, and can be determined in the laboratory with reasonable specimen sizes. An experimental technique, originally developed for concrete, can be used to determine the tension-softening curves of rocks.

1. INTRODUCTION

STANDARD methods are available for determining plane strain fracture toughness of metallic materials[1]. However, many rocks are quasi-brittle and the nature of fracture process zones in rock is different from plastic zones in metals[2, 3]. Thus the direct application of such standards to rock may not be acceptable. Initial steps toward standardization for fracture toughness determination of rock have been made[4]. Such a test method should, in addition to conforming to specific material behavior, utilize the geometry of a typical rock core for the test and require a minimum amount of specimen machining. Since rocks are relatively weak in tension, tests should preferably be done with compressive loadings where tensile fractures are induced. The short rod specimen proposed by Barker[5], burst cylinder specimen proposed by Clifton *et al.*[6], round bend bar specimen introduced by Ouchterlony[7, 8], and semi-circular bend (SCB) specimen proposed by Chong *et al.*[9, 10] satisfy most of these requirements. Modified ring tests[11] which minimize the process zone length have also been proposed. However, considerable machining is required for rings. A future standard method for determining fracture toughness of rock materials may make use of some of these specimen types from a single core of material[10] for a complete characterization in the material's principal directions.

Mode I fracture is important for the understanding, design, and optimization or rock fracture processes. For rocks with small grains, such as oil shale, the elastic fracture toughness, K_{Ic} has been found to be a useful parameter for the characterization of intact rock with respect to its resistance to crack propagation and as an index of fragmentation processes in the analysis of fracturing[12–15].

Geomaterials, including concrete, ceramics, rocks and possibly ice, generally exhibit tension-softening behavior during fracturing. While K_{Ic} fracture toughness tests may be meaningful for some metals and other materials with small-grains, they are generally inadequate for toughness testing of geomaterials which exhibit quasi-brittleness, resulting in large process zones[3, 16] at crack tips. Existing experimental data based on classical LEFM fracture toughness tests indicate that the apparent toughness increases with respect to crack lengths for limestone, concrete, and other geomaterials. A valid K_{Ic} test generally requires impractically *large* specimen sizes. The minimum dimension in concrete specimens may be as much as three feet.

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Concrete and rock fracture resistance has been mostly characterized in terms of toughness. In this paper, we will review observations of process zones, specimen size dependence of fracture toughness determined by LEFM theory in the laboratory, and for rocks, the discrepancies between toughness estimation in the laboratory and in the field. Such discrepancies are observed both in mode I tension and in mode II shear. Next, we present a tension-softening constitutive model which is consistent with several macro mechanical and micro-mechanical experimental observations. This model is shown to predict the size dependence mentioned above. A non-linear J-integral based technique is then proposed for accurate determination of this tension-softening curve and fracture toughness using laboratory size rock specimens. Experience in the use of this technique in testing mortar and fiber-reinforced mortar is briefly described. Finally some preliminary results on tension-softening behavior of basalt rock are presented.

2. FIELD VS LABORATORY OBSERVATIONS

It has been observed that the propped fracture length estimated using pressure build-up tests and/or production data is often much less than predicted by hydraulic fracture simulators[17, 18, 19]. *In-situ* toughness determined from field pressure data by Shlyapobersky[20] suggests that the *in-situ* values can often be as much as one to two orders of magnitude higher than LEFM based laboratory results. Underestimation of fracture toughness based on linear elastic fracture mechanics applied to laboratory size rock specimens have been known for some time[12]. Figure 1 shows the increasing apparent toughness of rock with a planar dimension of the specimens, in this case the crack length. Solid curves are size-dependent K_Q-predictions based on cohesion model of Li and Liang[22], reported in Li[23]. It is observed that the steady state value or the true K_{Ic} could be achieved only when the specimen size becomes impractically large.

In mode II, Li[23] reported four orders of magnitude difference between fracture toughness data 'measured *in-situ*' from observations of seismic events ($\sim 10^6$ J/m^2) and that measured in the laboratory based on LEFM ($\sim 10^1$–10^2 J/m^2,[24]). However, in this case part of the discrepancy may be due to the very different dimensions of heterogeneities associated with the linking up of diagonal tensile cracks in the shear rupture process sometimes running up to hundreds of km size scale in comparison to the much smoother rupture in laboratory experiments.

Indeed, the size dependence of apparent fracture toughness is common in any quasi-brittle material (sometimes known as non-yielding material). For example, Francois[25] pointed out that the apparent toughness of concrete increases with the dimension of the specimen used in the test, as shown in Fig. 2. "Classical" toughness of concrete and cement are often reported in text books (e.g. Ashby and Jones[26]) as having a value of approximately 0.2 MPa $\sqrt{m}$. This reflects the lower limit of the data shown in Fig. 2 for small specimens. Based on measurements of tension-softening curves using a technique to be described in Section 7, Li *et al.*[22] reported a mortar toughness

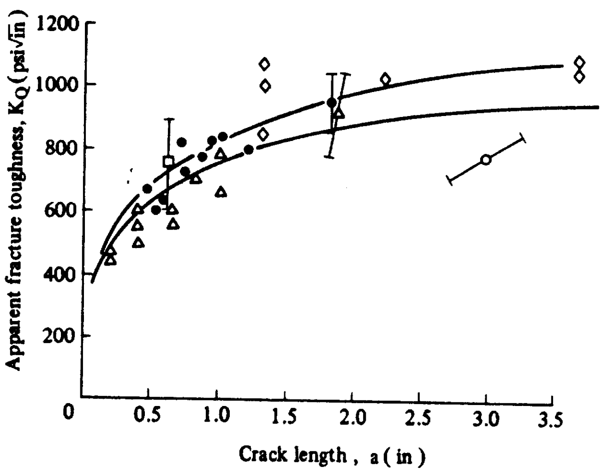

Fig. 1. Increasing apparent fracture toughness K_Q with crack length of Indiana Limestone specimens (from Ingraffea *et al.*[21]).

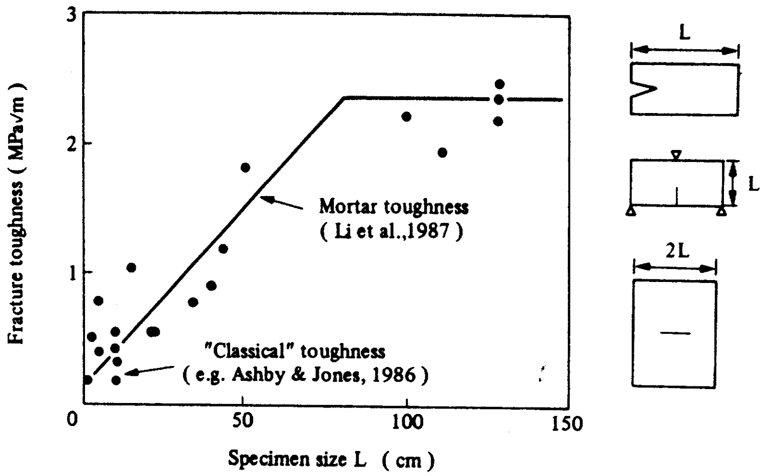

Fig. 2. Various results of the fracture toughness K_Q taken from the literature as a function of the size of the specimens (from Francois[25]).

of 1.5 MPa $\sqrt{m}$. The toughness for concrete based on such a technique may be expected to be even higher, and should coincide with the upper limit of the data (about 2.4 MPa $\sqrt{m}$) shown in Fig. 2. This demonstrates that measurements based on classical LEFM technique may underestimate the true toughness of certain geomaterials by as much as an order of magnitude, especially for those with large scale heterogeneities, and using typical laboratory size specimens.

These observations of size dependence of apparent fracture toughness in laboratory tests is consistent with the findings of Shlyapobersky[20], in that the *in-situ* field measurements could be interpreted as experimental results from 'specimens' many times larger than typical laboratory size specimens, and hence also result in a toughness value many times larger than that obtained from a laboratory test based on LEFM.

The question remains as to how laboratory tests could produce a toughness value closer to the *in-situ* true fracture toughness. We can either build a huge laboratory and test huge specimens; or we can abandon the concept of linear elastic fracture mechanics (LEFM). We propose to develop an experimental technique based on non-linear elastic fracture theory which not only extracts the fracture toughness of rocks accurately but also provides additional information on the tension softening behavior. Such information is often extremely important in analysing the failure stability of a *rock mass*.

It should be pointed out that the size effect in this discussion is related to the use of LEFM in analysing fracture toughness test data. There are, of course, other types of size effects related to other material properties. An important one in rocks is the effect of the greater probability of larger flaws occurring in a larger volume of material. This size effect, however, is reasonably well understood on the basis of flaw distribution in a quasi-brittle material (see, e.g. Ashby and Jones[26], Baecher and Einstein[27], and Einstein *et al.*[28]).

3. TENSION-SOFTENING BEHAVIOR IN ROCK

By tension-softening, we mean the gradual decrease in tensile load bearing capacity as material separate across an eventual failure surface (it is easier to think of this for a uniaxial tension specimen). The micromechanisms responsible for tension-softening could be grain sliding, micro-crack linking and possibly frictional interlocking. This tension-softening behavior is responsible for the development of the crack tip fracture process zone.

Stable uniaxial tests of Barre granite, Berea sandstone, Valdor limestone and Tennessee marble have been carried out by Peng[29]. A set of the results for Barre granite are shown in Fig. 3(a). The corresponding tension-softening curve can be deduced by assuming elastic unloading from the softening regime, and is shown (for the 2.2×10^{-4}/s strain rate data) in Fig. 3(b). The limited data set from[29] suggests that these diverse rock types exhibit quite different tension-softening behavior, depending on the material microstructure (and loading rate).

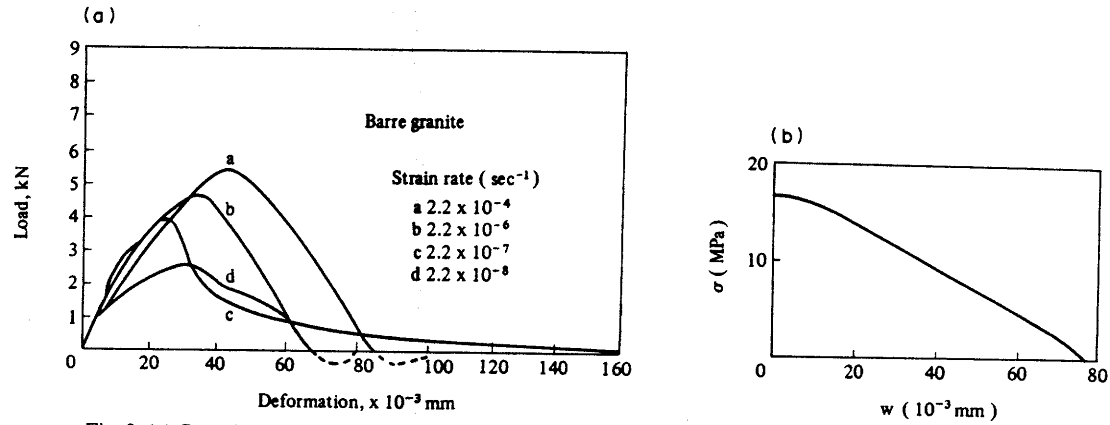

Fig. 3. (a) Complete load–deformation curves for Barre granite tested under various strain rates in uniaxial tension (from Peng[27]). (b) Tension-softening curve for Barre granite deduced from load–deformation curve of uniaxial tension test shown in (a) (strain rate = 2.2×10^{-4}/s).

To test the specimen size independency and for a cross-check against the Stress Intensity Factor (SIF) method, applied to semi-circular specimens[9, 10] illustrated in Fig. 4, the J-integral method is used for two basalt rock specimens with slightly different crack lengths (a_1 and a_2). Figure 5(a) shows the load vs load line displacement (LLD, measured by LVDT) for these two specimens. Tension softening behavior is obvious for both curves. Referring to Fig. 5(a), with subscripts 1 and 2 indicating the two slightly different specimens, and, modifying eq. 4 (to be presented in Section 7),

$$J_c = \left(\frac{A_1}{B_1} - \frac{A_2}{B_2}\right)\frac{1}{\Delta a} = 600 \text{ N/m} \tag{a}$$

where B_1 and B_2 are the net thicknesses; A_1 and A_2 are areas under each curve; Δa is the difference between the crack lengths a_1 and a_2.

Figure 5(b) shows J vs LLD, with J_c approaching 600 N/m. Figure 5(c) illustrates the tension softening curve, the area under which equals J_c (see Section 7).

Based on measurements of sound velocities in a direction perpendicular to the crack, the average Young's modulus is computed[35],

$$E = 1.665 \times 10^{10} \text{ N/m}^2. \tag{b}$$

Since the J_c given in eq. (a) is size independent, fracture toughness can be given as

$$K_{Ic} = (EJ_c)^{1/2} = 3.2 \text{ MPa } \sqrt{m}. \tag{c}$$

Based on SIF method[35], using the average of the two specimens

$$K_{Ic} = 3.3 \text{ MPa } \sqrt{m} \tag{d}$$

which is very close to the J-based K_{Ic} given by eq. (c). Further testing is under way.

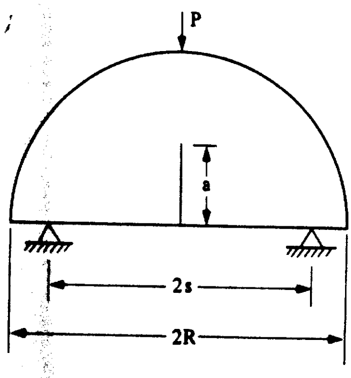

Fig. 4. Semicircular specimen.

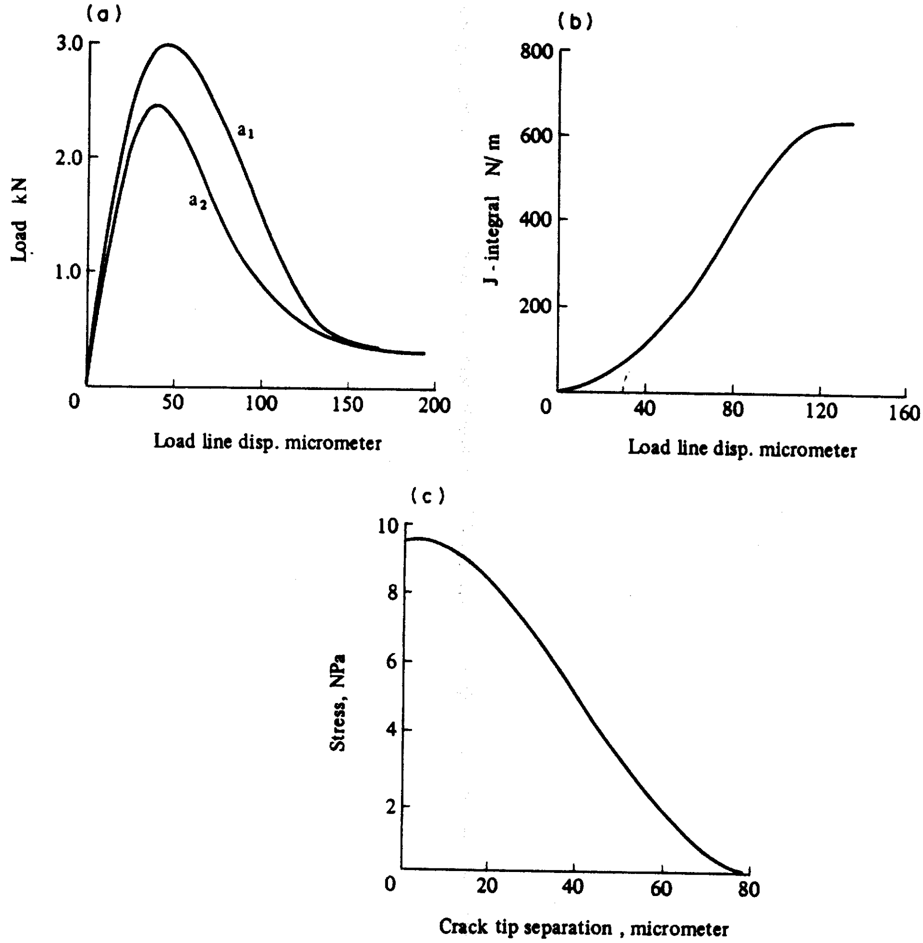

Fig. 5. (a) Load–deflection curves of basalt. (b) Values of J vs LLD. (c) Tension softening curve.

More experimental information than for tension softening is available on slip weakening behavior. Rice[30] reported slip weakening curves for granitic rocks. Stable or semi-stable rock joint tests have been performed by a number of research groups. Slip weakening behavior is a component of shear behavior which is of interest both regarding cracks propagating in mode II or mixed modes and regarding rock joints.

4. THEORETICAL AND EXPERIMENTAL WORK ON COHESION FRACTURE MODELS AND PROCESS ZONE DEVELOPMENT

The link between tension-softening and fracture behavior is provided (amongst others), by Li and Liang[22] who analysed the process zone development of a simple center cracked panel made of a quasi-brittle material. They demonstrate that the development of a long process zone is responsible for the inapplicability of LEFM, and that for such a material, an indiscrete use of LEFM will lead to an underestimate of the true fracture toughness. Li[23] used this model to predict the increase of rock fracture toughness as a function of crack length, shown as the solid curves in Fig. 1.

Presence of a long process zone at a crack tip has been repeatedly observed in various quasi-brittle materials. For example, using acoustic emission sensors, Labuz *et al.*[31] tracked development of the process zone in double cantilever beam specimens of Charcoal and Rockville granite (Fig. 6). They concluded that the length of the ligament process zone could form a substantial portion of the effective crack. Many other investigators[3] have found similar results.

The tension softening curve is a very useful property of quasi-brittle materials. Apart from providing size-independent fracture parameters, it may also be used in numerical fracture

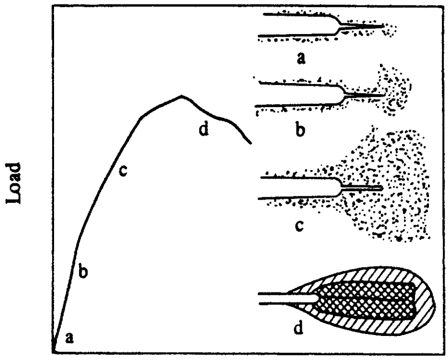

Fig. 6. Micro-cracking identified by acoustic emission, and process zone development (from Labuz *et al.*[31]).

simulations where LEFM is not applicable. Such situations arise, e.g. when fractures run into geometric or material boundaries in the rock mass, when crack and/or joints interact, and when cracks are forming at a stress concentrator. In those situations, the small scale yielding condition required by LEFM may be violated.

4.1. *Tension softening curve as a non-linear fracture parameter*

Using the J-integral with a contour around the process zone in which tension-softening occurs, Rice[34] showed that the area under the tension-softening curve can be related to the critical energy release rate for imminent fracture. Thus if the tension softening curve can be determined experimentally, then it is possible to deduce the critical energy release rate and the fracture toughness. Note that in this scheme, the experimental portion does not involve the use of LEFM theory and hence, there is usually no specimen size requirement as in the case of a classical fracture toughness test.

The tension-softening curve can also be applied directly in numerical analysis as a constitutive relation between tensile stress and opening for all points along the crack line. This is indeed the method employed by Li and Liang[22] to track the extension of the fracture process zone in their numerical model. More recently, Chong and his associates are investigating 3D process zones in a NSF sponsored project.

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4.2. *Experiments in tension-softening measurements*

Direct tension tests for quasi-brittle materials are usually problematic when it is necessary to carry the test stably into the post-peak regime. In addition, the problem of instrumentation (location of displacement transducers or strain gages across potential crack planes) and the sensitivity of slight loading eccentricity on the test result make the direct tension test difficult to carry out.

In the last several years a method has been developed at the Center of Advanced Construction Materials at MIT[32, 33] for the indirect determination of tension-softening curves for quasi-brittle materials. The theoretical basis is the J-integral and the concept of tension-softening, and has been described in detail in Li *et al.*[32]. To provide the perspective, the J-integral test method for metal has now been well defined and accepted, and has had a strong impact on the toughness testing of ductile metals. Our work is aimed at developing, along parallel lines the theoretical basis and experimental technique for toughness evaluation of quasi-brittle materials which do not plastically yield, but nevertheless have a process zone large enough to invalidate LEFM. The testing technique has been applied to mortar and steel fiber reinforced concrete[32, 33] using a compact tension specimen and a 4-point bend specimen. The results have been compared to those from stable direct tension tests in similar materials, and have been shown to be relatively consistent. It may be necessary, however, to carry out an independent direct tension test just to obtain the tensile strength, as the indirect test often produces some scatter of this value from test to test[33].

4.3. *Theoretical basis*

Rice[34] showed that the path-independent J-integral is

$$J = \int_{\Gamma} [W\,\mathrm{d}y - T_i(\mathrm{d}u_i/\mathrm{d}x)\,\mathrm{d}s] \tag{1}$$

where W is the elastic strain energy density, T_i is the traction vector, u_i is the displacement vector, and other quantities are defined in Fig. 7.

For a material which exhibits localized inelastic deformation on a plane ahead of the crack tip the J-integral in (1) reduces to[34],

$$J = -\int_a^L \sigma(x)(\partial\delta/\partial x)\,\mathrm{d}x \tag{2}$$

where $\sigma(x)$ and $\delta(x)$ are the normal stress and opening displacement at the point x in the process zone. The upper integral limit L is measured to a point x ahead of the physical crack tip where $\delta(x) = 0$. Changing variables from x to δ, and since σ is a single valued function of δ (i.e. for each δ, there corresponds a unique value of σ in the σ–δ relation, Fig. 8),

$$J(\delta) = \int_0^{\delta} \sigma(\delta)\,\mathrm{d}\delta. \tag{3}$$

A critical value of $J = J_c$ is reached when the separation δ at the physical crack tip reaches δ_c. In the special case where the process zone size $(L - a)$ is much smaller than all other planar dimensions in the problem, J_c and the critical energy release rate G_c coincide[34]. The σ–δ curve, however, does not need to observe the small scale yielding condition of LEFM and is thus a more fundamental material property than G_c. An approximate procedure to find $J(\Delta)$ is to use two specimens identical in every respect for a small difference in notch length $(a_2 - a_1)$. The change in energy may be interpreted as the area between the P–Δ curves for each of the two specimens, i.e.

$$J(\Delta) = (1/B)[\text{Area }(\Delta)/(a_2 - a_1)]. \tag{4}$$

Since a_1 and a_2 are pre-cut (or cast) notch lengths, there should be no difficulty calculating $J(\Delta)$. It may be expected that $J(\Delta)$ increases from zero and asymptotically approaches J_c.

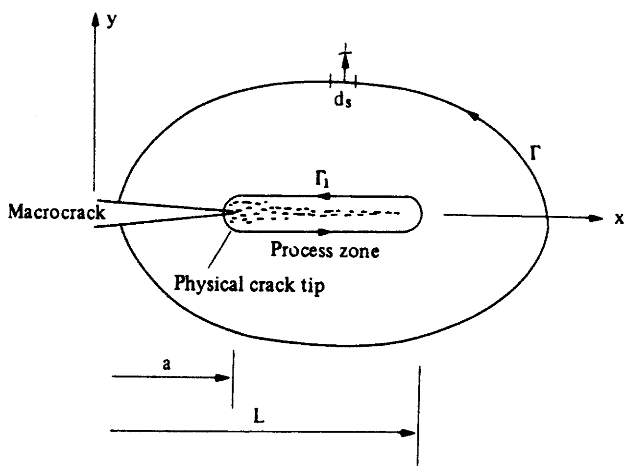

Fig. 7. J-Integral contours around the crack tip.

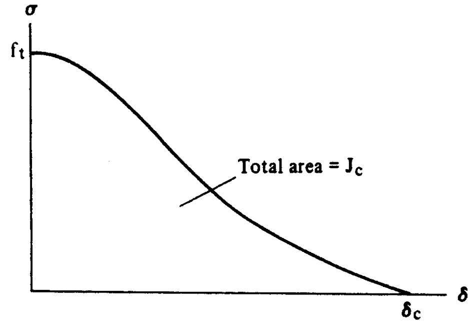

Fig. 8. Tension-softening curve.

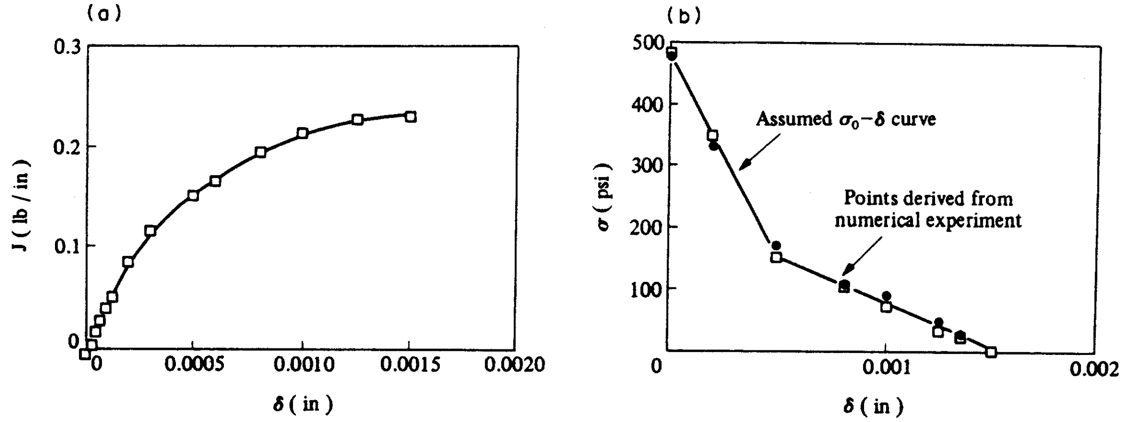

Fig. 9. (a) Derived J vs δ curve. (b) Comparison of assumed and derived stress-separation curves.

To deduce the tension-softening relation, eq. (3) may be utilized, since by differentiation,

$$\sigma(\delta) = \partial J(\delta)/\partial\delta \tag{5}$$

and combining with (4)

$$\sigma(\delta) = \{1/[B(a_2 - a_1)]\}[\partial \text{ Area } (\Delta)/\partial\delta]. \tag{6}$$

To use eq. (5) effectively, Δ and δ must be correlated by recording the load line displacement and the opening at the crack tip during the loading process. Thus the experimental output required to obtain $\sigma(\delta)$ are a set of P–Δ and P–δ relations for each type of specimens with different notch lengths. Details of instrumentation can be found in Li *et al.*[32].

4.4. *Verification of testing procedure*

An independent verification for our testing procedure was provided by A. Hillerborg (private communication, 1985). He employed his fictitious crack model in a finite element scheme to simulate the load–deformation curves and load–crack tip opening curves of a pair of three point bend specimens of slightly different notch lengths. He used an artificial bilinear curve as input for the tension-softening behavior in the material ahead of the notches. The objective of the exercise is to extract this same curve using our procedure[32] and his numerically derived 'test' results. If the J-integral based test technique is theoretically sound, then he should get a predicted tension-softening curve overlapping the bilinear curve he input into his finite-element program. The result is most encouraging. A similar exercise was carried out by Reyes (1987) based on a boundary element method combined with fracture propagation capability. The numerically derived J–δ and σ–δ curves are shown in Fig. 9(a) and (b). Comparison between the input σ–δ data and the derived σ–δ information shows that they essentially overlap one another. The good agreement supports the J-integral procedure proposed by Li *et al.*[32] for experimentally deducing the tension-softening curve.

5. CONCLUSION

The theoretical work by Li and Liang[22] provides the physical basis for understanding the expected underestimation of fracture toughness of quasi-brittle materials using LEFM theory. The application of the J-based tension-softening test technique[32, 33], adapted to the semicircular specimens, developed by Chong *et al.*[9, 10], facilitates the fracture characterization of rock cores and other cylindrical specimens. We have now a methodology in hand to obtain fracture test results which are *size independent*. We suggest that the application and refinement of this test methodology to rocks may eliminate certain discrepancies between model predictions and field observations in rock mass behavior and other applications, such as overpressures in hydraulic fracturing processes. Once this method is established, an updated data base of rock fracture toughness data can be obtained for use by engineers.

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