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NUMERICAL ANALYSIS OF THE J-BASED FRACTURE TESTING TECHNIQUE FOR BRITTLE MATRIX COMPOSITES

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ABSTRACT

This paper presents a numerical analysis performed to verify the newly extended J-based fracture testing technique for determining the tension-softening relation in brittle matrix composites. The original J-based technique has been well established for quasi-brittle materials where the fracture process is primarily dominated by the formation of fracture process zone and the contribution of the crack tip toughness is negligibly small. Recently, the J-based technique has been further developed to cover a more general case, *i.e.* a material in which the crack tip stress singularity coexists with the fracture process zone. In the previous experimental investigation, the extended J-based test procedure was applied to brittle matrix composites with the crack tip toughness.

In this study, computer simulations using a boundary element method have been performed to verify numerically the newly extended J-based technique. The numerical results demonstrate the validity of the J-based technique.

INTRODUCTION

Brittle matrix composites have been the focus of substantial research effort in recent years. The major driving force for brittle matrix composite development is the potential of utilizing ceramics in load-carrying structural components with greater mechanical and thermal reliability. A feasible approach for improving mechanical properties is through the enhancement of the fracture toughness. It has been established that the primary mechanism for the toughness improvements is due to the formation of the fracture process zone behind the crack tip.

A Barenblatt type cohesive model [1, 2] has been frequently employed to describe the development of the fracture process zone in brittle matrix composites [3, 4]. Development of the fracture process zone is governed by a relationship between the traction $\sigma(x)$ acting at a point x on the crack plane and the corresponding separation distance of the crack faces, $\delta(x)$. The σ–δ relation is referred to as the tension-softening relation. By definition, the relation can be obtained from a direct tension test. In most instances, the σ–δ relation controls the overall fracture behavior of the composite. For example, if σ decays rapidly at small δ, a resemblance of small scale yielding LEFM type behavior is attained. If σ decays slowly, the Dugdale type yielding behavior results. In between, the material exhibits quasi-brittle behavior often described by the R-curve. The σ–δ relation also provides useful fracture parameters for material characterization such as composite tensile strength and fracture toughness. In addition, it could be used for numerical simulations of crack formation and propagation in structures made of the composite. Therefore, further development in micro structural design of brittle matrix composites and mechanical performance prediction of composite structural parts calls for a reliable testing technique for determining the σ–δ relations.

Li [5] and Li *et al* [6] have proposed a novel J-based fracture testing technique to determine the tension-softening relation originally for characterizing the concrete fracture behavior. The testing technique has the advantage of requiring only a simple stroke controlled loading machine and is relatively stable in comparison with direct uniaxial tensile tests. It has been applied by several researchers to a number of quasi-brittle materials [7, 8, 9], in which the fracture process is primarily dominated by the formation of fracture process zone and the contribution of the crack tip singularity is negligibly small. It has been shown, however, that in current advanced brittle matrix composites, the term of crack tip singularity cannot be neglected, and the bridging toughness, which is due to the fracture process zone, is of the same order of magnitude as the crack tip singularity term. From this point of view, the J-based testing technique has been recently extended in order to explicitly account for the crack tip singularity while considering the fracture process zone[10, 11]. The testing procedure was applied to fiber reinforced foam glass and high strength concrete composites. It was shown that the tension-softening curve determined by the extended J-based fracture testing procedure was close to that obtained from uniaxial tension tests for the brittle matrix composites.

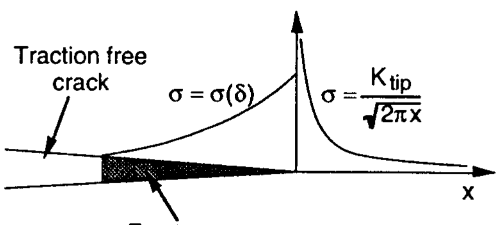
The objective of this research is to verify numerically the extended J-based testing procedure. For the numerical analysis, a boundary element method was developed to take into account the crack tip singularity as well as the formation of the fracture process zone. The fracture behavior of compact tension (CT) specimens was simulated based on the tension-softening model with the crack tip singularity. The input data for the analysis were the crack tip toughness, the tension-softening curve and the elastic moduli of the material. In the analysis, the magnitude of the crack tip toughness was varied to simulate different brittle matrix composites. The newly derived testing technique was applied to the numerical data in order to determine the tension-softening relation and to compare the deduced curve with the input curve.

OUTLINE OF EXTENDED J-BASED TESTING METHOD AND COMPUTER SIMULATION PROCEDURE

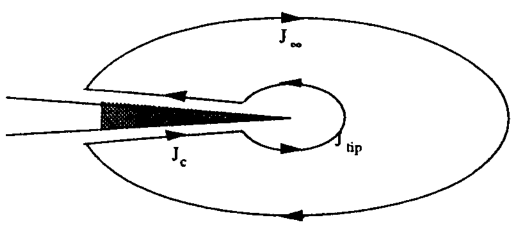
This section provides a brief review of the principle of the J-based technique, followed by the description of numerical simulation procedure. A schematic of stress distribution around the crack tip is presented in Figure 1(a) for a material where the crack tip singularity coexists with the fracture process zone. For the closed contour shown in Figure 1(b), the J-integral path independent property [12] requires

$$J_\infty + J_c + J_{tip} = 0 \qquad (1)$$

The J_∞ term represents the energy release rate associated with far-field loading, and contains information on the specimen geometry. The J_{tip} is the crack tip singularity term. Finally J_c is the energy consumed by the development of the fracture process zone. Applying the J-integral analysis to the terms J_{tip} and Jc for the material with the crack tip singularity and fracture process zone, Equation (1) can be expressed by



(a) schematic of stress distribution around the crack tip



(b) J-integral contour

Figure 1 Principle of J-based testing technique

$$J_\infty = \int_0^{\delta_t} \sigma(\delta)d\delta + \frac{K_{tip}^2(1-\upsilon^2)}{E} \tag{2}$$

Where δ_t is the crack opening displacement measured at the original crack tip, K_{tip} is the crack tip stress intensity factor, and E and υ are the Young's modulus and Poisson's ratio for the composite, respectively. Differentiating Equation (2) with respect to δ_t, the σ–δ curve may be determined from

$$\sigma(\delta_t) = \frac{\partial}{\partial \delta_t}\left[J_\infty(\delta_t) - \frac{K_{tip}^2(1-\upsilon^2)}{E}\right] \tag{3}$$

Thus if J_∞ and K_{tip} can be determined experimentally, then the σ–δ relation can be derived from Equation (3).

In Figure 2, the flow of the testing procedure based on the above theoretical foundation is presented. J-integral value can be evaluated using load-displacement curves obtained from two specimens with different crack length as illustrated in Figure 2. Necessary data for the J-based method are load, P, load-line displacement, δ_L and crack tip displacement, δ_t. The value of J-integral for a given value of δ_L is calculated using the following equation.

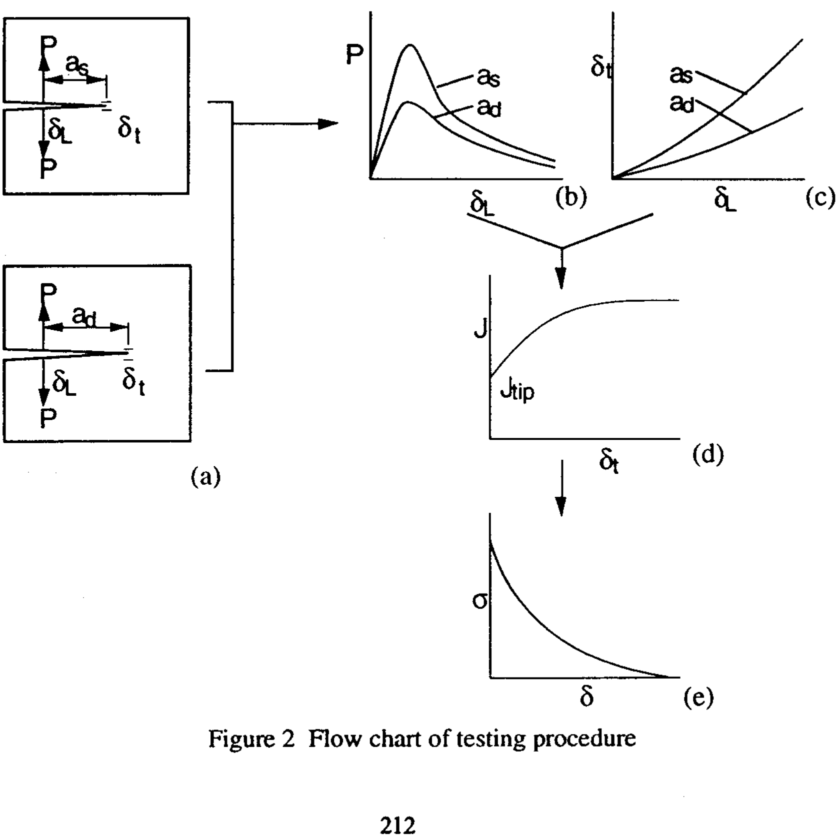


Figure 2 Flow chart of testing procedure

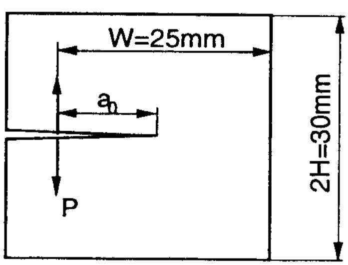
$$J_\infty = \frac{1}{a_d - a_s}\int_0^{\delta_L}(P_s - P_d)d\delta_L \qquad (4)$$

Where P is load per unit specimen thickness. The subscripts, s and d refer to the short and large notch lengths, respectively. Based on a set of $P-\delta_L$ and $\delta_t-\delta_L$ relations for each specimen with different notch lengths, the relation between J and δ_t is obtained as in Figure 2(d). Figure 2(e) shows the $\sigma-\delta$ curve deduced from the slope of the $J-\delta_t$ curve.

In order to examine the validity of the above testing procedure, numerical analyses were conducted using a boundary element method (BEM). Compact tension (CT) specimen, as shown in Figure 3 was used in the numerical analysis. Figure 4 shows the boundary element mesh used. Considering the symmetrical geometry of the CT specimen, the upper half was discretized into 84 elements. The plane strain condition was assumed. The specimen to be analyzed was assumed to behave elastically everywhere except inside the fracture process zone, which followed the prescribed $\sigma-\delta$ curve. The development of the fracture process zone was simulated by the separating of the nodes ahead of the crack tip, and by shifting the current crack tip node with the incremental step of one mesh size. The crack tip node was released when the stress field ahead of the crack tip satisfied the condition dictated by the crack tip singularity,

Table 1 Toughness values used

J_{tip}/J_c	0	1/5	1	3	5
K_{tip}	0	5	11.2	19.4	25.0
J_{tip}	0	83.3	417	1251	2083
J_c	417	417	417	417	417



(a) compact tension specimen

(b) initial crack length used

No.	a_0 (mm)	a_0/W
a_1	12.00	0.48
a_2	12.25	0.49
a_3	13.00	0.52
a_4	15.00	0.60
a_5	19.00	0.76

Figure 3 Specimen configuration used

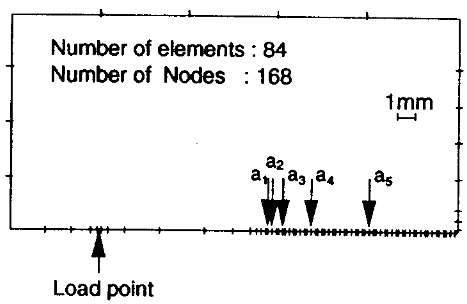


Figure 4 Boundary element mesh

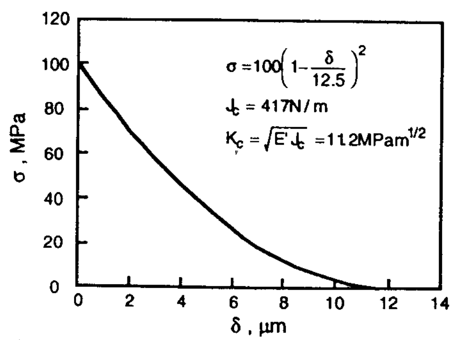


Figure 5 Tension-softening curve used

K_{tip}. Thus, the simulation was performed to maintain the crack tip singularity during the crack advance. A corresponding value of load and displacement was obtained by the BEM which satisfies the crack tip singularity as well as the tension-softening relation for the fracture process zone.

As described above, a pair of specimen with different notch length are used to determine the J-integral. Specimens with five different initial crack length were analyzed. The σ–δ curve used in the simulation is shown in Figure 5. The J_c value calculated from the area under the curve is 417 $MPam^{1/2}$. In the analysis, the magnitude of the crack tip toughness, J_{tip} was varied, as shown in Table 1. Hereafter, J_{tip} refers to the crack tip toughness used for the condition of the node release at the crack tip. The ratio J_{tip}/J_c was in the range of 0 to 5. The Young's modulus was 300 MPa and the Poisson's ratio was set to be 0.25.

RESULTS AND DISCUSSION

Because the extended J-based method intends to extract the σ–δ relation for materials with non-negligible crack tip toughness, the simulation procedure should be able to accurately treat the crack tip stress singularity in order to verify the J-based method. Therefore, the validity of the numerical simulation is checked with regard to the crack tip singularity. It is well known that the crack opening displacement in the vicinity of the crack tip is proportional to the square root of the distance from the crack tip for linear elastic solids. The crack opening displacements computed using the developed BEM are plotted against the square root of the distance from the crack tip in Figure 6. The solid lines show the theoretical distribution calculated from the crack tip singularity K_{tip} according to the linear elastic fracture mechanics. It can be seen that the simulation results agree well with the theoretical results irrespective of the different crack tip toughness ratio, J_{tip}/J_c. This agreement was obtained for any crack growth stage. The above evidence demonstrates that the crack tip stress singularity is well simulated in the numerical procedure. Thus, the present numerical procedure can be used to verify the validity of the J-based method.

Figure 7 (a) and (b) show P–δ_L curves for all the initial crack lengths obtained from the numerical simulations. Figure 7 (a) shows the simulation results for the crack tip toughness ratio, J_{tip}/J_c=0 and the results for J_{tip}/J_c=3 are shown in Figure 7 (b). For J_{tip}/J_c=3, the maximum load is higher than that for J_{tip}/J_c=0 because of the presence of the crack tip toughness. P–δ_L curves for the different initial crack lengths finally join with each other. Figure 8 shows typical example of δ_L–δ_t curves for the initial crack length of a_0/W=0.48. For J_{tip}/J_c=0, the δ_L–δ_t curve starts from the origin. In contrast, for J_{tip}/J_c=3, δ_t remains zero until δ_L reaches the load level corresponding to the matrix fracture toughness, J_{tip}.

$J-\delta_t$ relations obtained from the $P-\delta_L$ and $\delta_t-\delta_L$ data are shown in Figure 9 and deduced $\sigma-\delta$ curves are shown in Figure 10. J-integral value is calculated using a pair of load–displacement data for different initial crack lengths. It is seen in Figure 9 that the $J-\delta_t$ curves give a finite initial J value for $\delta_t=0$, for the case $J_{tip}/J_c=3$. The initial J value corresponds to the matrix toughness J_{tip}. In Figure 10, the $\sigma-\delta$ curve used as the input for the calculation is also shown in order to compare with the deduced $\sigma-\delta$ curves. It is noted that the deduced $\sigma-\delta$ curves agree with the input curve when the difference of initial crack length Δa is relatively small. This result supports the validity of the extended J-based method which takes into account the crack tip toughness.

It is seen that when the Δa becomes larger the deduced $\sigma-\delta$ curve significantly deviates from the input curve. For $J_{tip}/J_c=0$, the J-based method gives the same $\sigma-\delta$ curve as the input curve except for the largest Δa. The discrepancy observed for $J_{tip}/J_c=3$ is larger than that for $J_{tip}/J_c=0$. Thus, appropriate Δa for determining the $\sigma-\delta$ curve appears to depend on J_{tip}/J_c. In principle, Δa should be as close to zero as possible in order to evaluate the J-integral. Therefore, it can be considered that the error

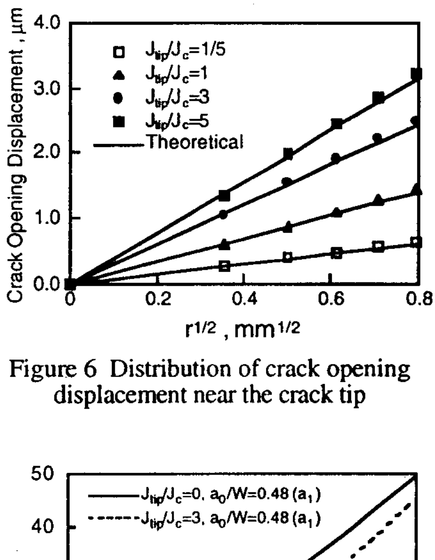
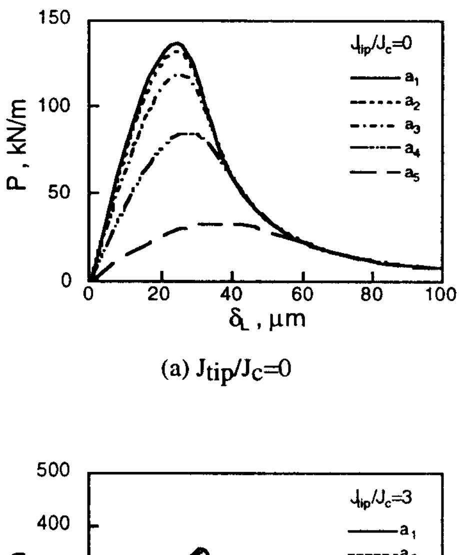


Figure 6 Distribution of crack opening displacement near the crack tip



(a) Jtip/Jc=0

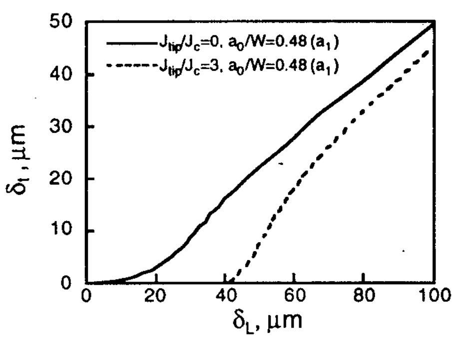
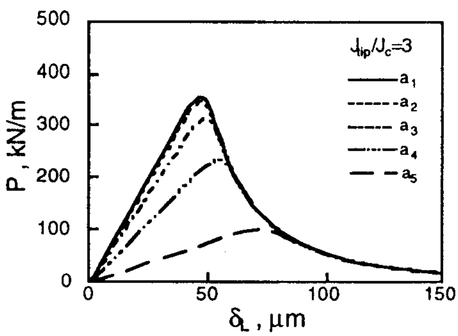


Figure 8 $\delta_L-\delta_t$ curves



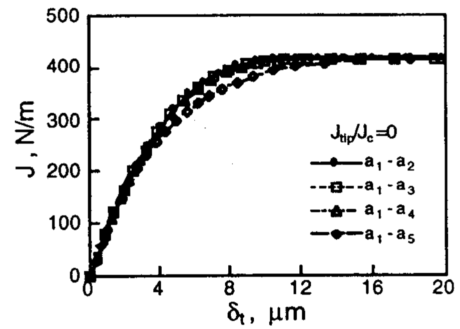
(b) Jtip/Jc=3

Figure 7 $P-\delta_L$ curves

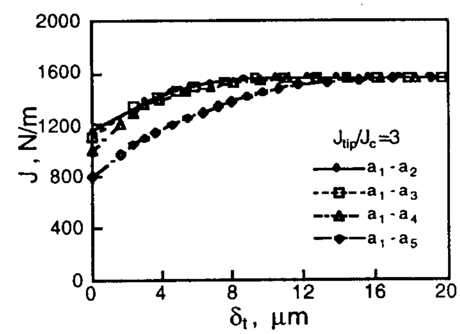
observed in Figure 10 is induced by the finite difference in initial crack lengths. In order to characterize the deviation with respect to Δa used in this study, the following error quantity is calculated:

$$\varepsilon = \int_0^{\delta_c} \left| \sigma_{\text{deduced}} - \sigma_{\text{input}} \right| d\delta / J_c \qquad (5)$$

Where $\sigma_{deduced}$ and σ_{input} are the deduced and input cohesive stresses, respectively. δ_c is the critical crack opening displacement when the cohesive stress becomes zero, and J_c is the critical J–integral value . The calculated error is plotted in Figure 11 as a function of J_{tip}/J_c. It is clear that the error increases with increasing Δa. In practice, experimental accuracy and material variability require the use of finite difference in initial crack lengths. It is quite usual that experimental scatter observed in brittle matrix composites is more than 10 %. Figure 11 allows us to determine the required Δa for obtaining appropriate σ–δ relation. The upper limit of Δa

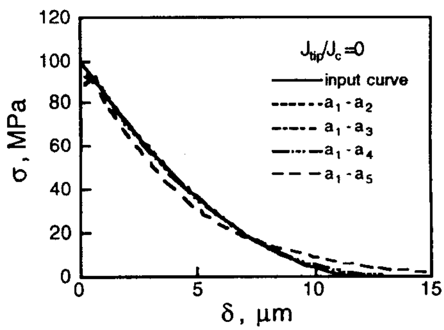


(a) Jtip/Jc=0

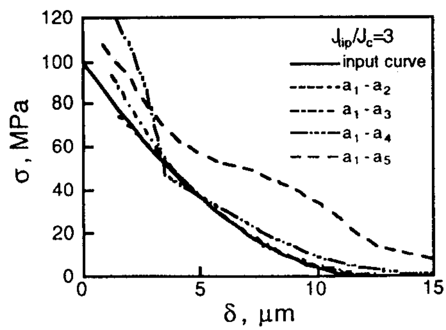


(b) Jtip/Jc=3

Figure 9 J–δ_t curves



(a) Jtip/Jc=0



(b) Jtip/Jc=3

Figure 10 σ–δ relations

under the given accuracy is plotted against J_{tip}/J_C in Figure 12. To obtain an estimate of J_{tip} and J_C, a preliminary test may be conducted. The matrix toughness or initial value of crack propagation resistance curve may give a measure of J_{tip}. Furthermore, the fracture energy obtainable from a single load–displacement curve [2] can be used as J_C. Thus, it is possible to select a suitable value of Δa using the preliminary experimental data and the upper limit curve shown in Figure 12. However, it should be mentioned here that the upper limit curve may depend on the elastic modulus, functional form of the σ–δ relation, and the specimen geometry in addition to the toughness ratio J_{tip}/J_C. Further study is needed to generalize the procedure for selecting a suitable Δa for brittle matrix composites .

CONCLUDING REMARKS

Numerical analyses were conducted to verify the extended J-based fracture testing technique in brittle matrix composites. The testing procedure has been devised to determine the tension-softening relation of a material which has matrix toughness comparable to bridging toughness induced by the fracture process zone. The numerical results obtained from boundary element analyses demonstrated the validity of the extended J-based fracture testing procedure. It was also shown that the difference of initial crack length should be reasonably small to obtain the J-integral and to determine the tension-softening relation accurately based on load-displacement records from a pair of fracture specimens. Using the numerical results, a procedure for selecting an acceptable difference in initial crack length was presented.

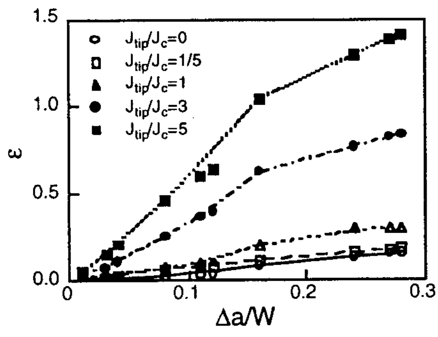


Figure 11 Error of the deduced σ–δ relations

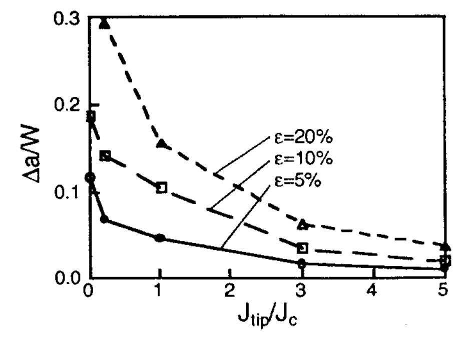


Figure 12 Upper limit of Δa

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