Ceram. Eng. Sci. Proc. 10[9–10] pp. 1164–1178 (1989)

First-Cracking Strength of Short Fiber-Reinforced Ceramics

C. K. LEUNG AND V. C. LI

Dept. of Civil Engineering
Massachusetts Institute of Technology
Cambridge, MA

The relation between first-cracking strength and inherent flaw size of short fiber-reinforced ceramic (sfrc) are evaluated using a fracture mechanics approach. Such a relation can be employed to predict material reliability. Results of the present investigation shows that the material properties affecting sfrc behavior can be grouped into a single dimensionless term, L/u_c, that governs the behavior of sfrc. Sfrc with small L/u_c values behave similarly to continuous fiber composites while sfrc with large L/u_c behave similarly to ideally brittle materials (Griffith materials). L/u_c is thus an important parameter to be considered in the design of sfrc.

Introduction

Monolithic ceramics are brittle materials and usually fail abruptly without giving much warning in advance. Moreover, due to their brittleness, the ceramic strength is very sensitive to the size of flaw in the material. Ceramic parts made of the same material may thus have very different values of strength. Hence, ceramics are also referred to as materials of low reliability. There are several ways to improve the toughness of ceramics but the addition of fiber reinforcement is by far the most effective means.[1] Marshall et al.[2,3] have shown that continuous fiber reinforcements can greatly improve the reliability of ceramics as the sensitivity of first-cracking strength to flaw size is significantly reduced. (Note: The first-cracking strength is the applied tensile stress at which an inherent flaw will propagate unstably across the whole section of the material.) After first-cracking, provided the fibers are strong enough, bridging of the crack by fibers will allow the material to take further load. With increased loading, multiple cracks will form, accompanying some pseudo-plastic deformation of the material.[4,5] This pseudo-ductility will provide warning before final failure and also allow for stress redistribution to less severely loaded parts.

While continuous fiber ceramics have been shown to possess the desirable properties described above, their use has been limited to parts of relatively simple geometric shape, as it is very difficult and costly to construct continuous fiber reinforcing mesh of complex shapes. On the other hand, short fibers can be mixed with ceramic powders and formed into any shape by traditional powder compaction techniques. Whisker-reinforced ceramics are well known examples of short-fiber ceramics made in such a way. Also, injection molding techniques, which are well developed for polymeric composites, can be applied to short fiber ceramic composites to make parts of complex shape.

While short fiber-reinforced ceramics (sfrc) have a processing advantage over continuous fiber-reinforced ceramics (cfrc), an important issue to be resolved is whether sfrc possess similar desirable properties such as high reliability and multiple cracking. Intuitively, one would expect that discontinuous fiber composites may possess the desirable behavior of cfrc, depending on how long the discontinuous fibers are. One may then be able to have fibers short enough for convenient processing but still long enough to have composite behavior similar to that of cfrc. The purpose of this paper is to study the factors affecting the first-cracking strength of sfrc in order to determine a criterion for sfrc to have high reliability and pseduo-ductility. In the following, an analytical technique based on fracture mechanics will be presented. The fiber bridging stress vs crack opening relation for short fiber composites is derived. To obtain an analytical solution (rather than a numerical solution), an approximate crack profile is employed. The final results are then discussed with emphasis on the sensitivity of sfrc behavior to the various material parameters.

Fracture Mechanics Approach

The relation of first-cracking strength to flaw size can be obtained by a fracture mechanics approach. In this study, the flaw is assumed to be a penny-shaped crack in an infinitely large body under uniform tension (Fig. 1). The first-cracking strength, σ, is then the applied tension at which the sum of stress intensities at the crack tip due to σ and the bridging force in the fibers (which are of opposite signs) equals a critical value, K_c, i.e.,

$$K_c = K_{\text{applied load}} + K_{\text{fiber bridging}} \qquad (1)$$

K_c is the composite toughness at initial crack propagation. In this analysis, following Refs. 2 and 3, we take $K_c = (E_c/E_m)\,K_m$, where K_m is the matrix toughness. In this expression, the mechanism for the crack to bow between the fibers and eventually circumvent them is not considered. Rose[6] proposed that $K_c = [(sK_m^2 + rK_L^2)/(s + r)]^{1/2}$, where r is the fiber radius, s is the spacing between fibers, and K_L is the toughness contribution for circumventing the fiber. At this state, an explicit expression for K_L has not yet been obtained (although Fares[7] has recently studied the case for a crack circumventing a fiber when both the matrix and fiber have the same moduli, solution for the bimaterial case has not yet been obtained). Therefore, the way we calculate K_c here is probably the best approximation one can make at this time.

For a penny-shaped crack,

$$K_{applied\ load} = 2\,(c/\pi)^{1/2}\,\sigma \qquad (1a)$$

Considering the effects of both the applied stress and the fiber bridging stress on the crack, from Ref. 8,

$$K_{fiber\ bridging} = 2\,(c/\pi)^{1/2} \int_0^1 p[u(X)]\, X/(1-X^2)^{1/2}\, dX \qquad (1b)$$

where $X = x/c$ as shown in Fig. 1, $u(X)$ is half the crack opening at X, and $p[u(X)]$ is thus the fiber bridging stress as a function of the opening along the crack profile. To be able to solve eq. (1), it is necessary to know the relation between p and u. This is derived as follows.

For a Single Fiber

Analysis of fiber debonding for the case of aligned fibers (fibers lying perpendicular to the plane of the crack) with purely frictional fiber/matrix bond has been carried out.[2] The result shows that for continuous fibers embedded in the matrix, until fiber breakage, p is a monotonic increasing function of u, given by

$$p = \alpha\sqrt{u} \qquad (2)$$

where $\alpha = V_f[4\tau E_f(1+\eta)/r]^{1/2}$

and $\eta = E_f V_f / E_m V_m$

Here, u is one-half the crack-opening; τ is the frictional bond strength at the interface; r is the fiber radius; E_f, E_m: Young's modulus of fiber and matrix, respectively; and V_f, V_m: volume fraction of fiber and matrix, respectively.

In the case of short aligned fibers, p cannot exceed the stress at which the full length of fiber has debonded, i.e.,

$$p < p_c = 2V_f\tau l_c/r \qquad (3)$$

where l_c is the shorter embedded length of the fiber on either side of the crack.

The value of u corresponding to p_c is given by

$$u_{\alpha} = \tau l_c^2/[E_f r(1+\eta)] \qquad (4)$$

For $u > u_{\alpha}$, the fiber is gradually pulled out of the matrix and we assume a linear drop of stress from p_c to 0 when u increase from u_{α} to $l_c/2$. The complete p vs u relation is plotted in Fig. 2.

Average for All Fibers

To compute the average stress vs crack opening relation for the composite, the following assumptions are being made:

(1) The shorter embedded length of fiber is a uniform distribution from 0 to l, where l is one-half the fiber length.
(2) All fibers are pulled out (i.e., the fibers are short enough to avoid any fiber breakage from taking place).
(3) The stress vs crack opening relation for inclined fibers is the same as that for an aligned fiber.

Define u_c as the half crack opening equal to the u_{α} of a single fiber with embedded length of l. Then, for $u > u_c$ in the composite, all the fibers have been debonded to its end and are on the way of being pulled out. The average bridging stress is

$$p = (1/2)(\alpha\sqrt{u_c})(1 - 2u/l)^2/(1 - 2u_c/l) \qquad (5)$$

For $u < u_c$,

$p = (\alpha\sqrt{u})[1 - \sqrt{(u/u_c)}]$ {Contribution from Debonding fibers}

$+ (1/2)(\alpha\sqrt{u})[\sqrt{(u/u_c)} - 2u/l]$ {Contribution from fibers that are being pulled out}

$$= (\alpha\sqrt{u})[1 - (1/2)\sqrt{(u/u_c)} - u/l] \qquad (6)$$

A complete p vs u curve showing average fiber stress vs half crack opening for the composite is shown in Fig. 3. Two points may be noted. The maximum stress, p_{max}, is reached at a value of half crack opening u_p less than u_c. Also, beyond u_c, there is a quadratic drop of p with increasing u, rather than the linear drop for the case of a single fiber.

After obtaining eqs. (5) and (6) for the p vs u relation, in order to solve eq. (1), one still needs to know the crack profile (i.e., the relation between u and X). For a penny-shaped crack, based on integration of effects from both the applied stress and the fiber-bridging stress on the crack opening,[9]

$$u(X) = [4(1-\nu^2)c/E_c\sqrt{\pi}]\int_X^1 [1/\sqrt{(s^2 - X^2)}] \int_0^s \{[\sigma - p(t)]\, t\, dt/\sqrt{(s^2 - t^2)}\}\, ds \qquad (7)$$

where E_c and ν are the composite Young's modulus and Poison's ratio, respectively.

For each applied stress, u(X) can be obtained through iteration between eqs. (5), (6), and (7). However, this will involve a lot of numerical computations. To avoid tedious computations and to obtain an analytical expression relating σ and c, it is assumed that the crack profile in this case is the same as that due to a uniform bridging stress, i.e.,

$$u(X) = [2(1-\nu^2)/E_c\sqrt{\pi}]K_c[c\,(1-X^2)]^{1/2} \quad (8)$$

Then, by substituting eqs. (5), (6), and (8) into eq. (1), the relation between σ and c can be obtained directly through integration.

Analytical Results

The present analysis shows that σ, when normalized by σ_m, can be expressed in terms of three other dimensionless terms, c/c_m, u_c/l and L/u_c, i.e.,

$$\sigma/\sigma_m = F(c/c_m, u_c/l, L/u_c)$$

where

c = size of the flaw and l = maximum embedded length of the fiber = fiber length/2

$$\sigma_m = [1.5\pi^{1/2}K_c^2\alpha^2\beta]^{1/3}$$

$$c_m = [9K_c\pi/4\alpha^2\beta]^{2/3}$$

$$L = [(3/2)\pi^{1/2}K_c^2\beta/\alpha]^{2/3}$$

$$u_c = \tau l^2/E_f r(1 + V_f E_f/V_m E_m)$$

$$\alpha = [4V_f^2\tau E_f(1 + V_f E_f/V_m E_m)/r]^{1/2}$$

$$\beta = 2(1-\nu^2)/(E_c\pi^{1/2})$$

$$L/u_c = \{[3/2]\,K_c^2 r^2 E_f(1+\eta)(1-\nu^2)]/[V_f E_c \tau^2 l^3]\}^{2/3}$$

For most practical composite material systems, L/u_c range from 0.1 to 3 and u_c/l lies within 0.0005 and 0.015. Instead of giving the complicated expression for σ/σ_m vs c/c_m, the relation is plotted as normalized stress vs normalized flaw size curves in Figs. 4–6. In Figs. 4 and 5, L/u_c is fixed at 0.1 and 3, respectively, with u_c/l varying from 0.0005 to 0.015. For $L/u_c = 0.1$, the curves for various values of u_c/l are indistinguishable. For $L/u_c = 3$, curves for the various u_c/l values

still do not differ significantly from each other. However, if we plot the curves for various values of L/u_c while keeping u_c/l constant at 0.005, as in Fig. 6, it is obvious that composite behavior is very sensitive to the value of L/u_c. For low values of L/u_c, the behavior approaches that of continuous fiber composites. Actually, $L/u_c = 0$ (for l approaching infinity) corresponds to the case of a continuous fiber composite. For $L/u_c = 0.1$, the first-cracking strength for sfrc is above 90% that for cfrc with the same volume fraction of fiber.

Similar to cfrc, for sfrc with low L/u_c values, the first-cracking strength reaches a constant value after a certain flaw size. This happens when the increase in $K_{\text{applied load}}$ with flaw size is exactly offset by the increase in $K_{\text{fiber bridging}}$ associated with the enlargement of the bridging zone once the average stress in the most heavily loaded fibers (i.e., the fibers furthest away from the crack tip) reaches the applied stress. At this steady state cracking, the crack propagates under constant stress with part of the crack profile remaining flat (Fig. 7). The presence of this range of constant stress has important implications. If the inherent flaw size of the material happens to lie close to or within this constant stress range (i.e., $> c_m$ from Fig. 6) the reliability of the material will be very high. Moreover, since the first-cracking strength will eventually reach a constant plateau value with increasing flaw size, subcritical crack growth during service will not decrease composite strength to below this plateau value. Hence, if we use the plateau strength value for large flaw size as the first-cracking strength of composite in design, then although the actual material strength may still depend on inherent flaw size, it will never be less than the value used in design. To the material user, the material is then still perfectly reliable.

For high L/u_c, the first-cracking strength keeps on decreasing with flaw size. Steady state cracking cannot occur and, as flaw size increases, there is no plateau value for the first-cracking strength. As L/u_c becomes larger and larger, composite behavior approaches that for a perfectly brittle Griffith material. $L/u_c = \infty$ (when $V_f = 0$) actually corresponds to the case of a Griffith material. However, for L/u_c up to 3, the curves for sfrc still lies much higher than the Griffith curve and are dropping less steeply, implying improved reliability. Also, the lower the value of L/u_c, the flatter are the curves and the higher the material reliability.

There is a value of L/u_c at which transition from low L/u_c behavior (behavior similar to cfrc) to high L/u_c behavior (behavior similar to Griffith material though still with improved reliability) takes place. This transition value can be obtained as follows. Define c_{ssc} as the value of c at which steady state cracking starts and c_p as the value of c when the maximum half crack opening exceeding u_p (i.e., at the descending branch of the composite p vs u curve; see Fig. 3). If $c_{ssc} < c_p$, steady state cracking can take place. The transition is obtained by putting $c_{ssc} = c_p$. For $u_c/l = 0.005$ (as in Fig. 6), the transition value is $L/u_c = 0.388$. Actually, it ranges from 0.396 to 0.371 as u_c/l varies from 0.0005 to

Table I. c_{mc}/c_m vs L/u_c, Where c_{mc} Is the Inherent Flaw Size Beyond Which Multiple Cracking Can Occur

L/u_c	0.1	0.2	0.3	0.4	0.5	1.0	3.0
c_{mc}/c_m	0.075	0.25	0.6	4	36	60	41

0.015. As an average, one can take $L/u_c < 0.38$ to be a criterion for the design of composites with high reliability.

Multiple cracking is another desirable behavior commonly observed in continuous fiber composites. Multiple cracking will occur if the stress that can be supported by the fibers alone is higher than the applied stress at first-cracking. For sfrc, it requires the first-cracking strength to be lower than the maximum stress that can be carried by the fibers (p_{max} in Fig. 3). It can be shown the p_{max}/σ_m is approximately equal to $(1/2)\sqrt{(u_c/L)}$. Since the first-cracking strength depends on flaw size, there is a flaw size c_{mc} beyond which the normalized first cracking strength σ/σ_m is lower than $(1/2)\sqrt{(u_c/L)}$. Values of c_{mc}/c_m for L/u_c from 0.01 to 3 are tabulated in Table I. For practical composites, it is very unlikely that the inherent flaw size becomes much higher than c_m. Thus, for L/u_c greater than about 0.4 (i.e., composites with high L/u_c behavior), there is a very low probability for multiple cracking to take place. For composites with low L/u_c behavior, the probability of multiple cracking increases with decreasing L/u_c. Specifically, for $L/u_c = 0.1$, multiple cracking requires $c_{mc}/c_m > 0.075$ and this is satisfied for most practical composite systems.

Implications for Material Design

From the results presented above, it is obvious that composite behavior is insensitive to the parameter u_c/l. Hence, L/u_c is the governing parameter in the design of sfrc. A low value of L/u_c is desirable as this will result in a very reliable composite with failure likely to be preceded by multiple cracking. Thus, a low value of L/u_c can be taken as a criterion (among other criteria for strength and toughness) for the design of reliable and pseudo-ductile sfrc.

For a physical interpretation, L/u_c may be re-expressed as:

$$L/u_c = \{K_c^2/[V_f E_c \tau^2 l^3/((3/2) r^2 E_f (1 + \eta)(1 - \nu^2))]\}^{2/3}$$

Here, the term K_c^2 in the numerator denotes the fracture toughness as the crack starts to propagate, while the other terms (expressed collectively within the square brackets [] in the denominator) indicate the contribution of fiber bridging to the overall toughness of the composite. A small value of L/u_c is obtained if the contribution of fiber bridging to composite toughness is high compared with the initial toughness, i.e., the composite shows significant R-curve behavior. The improvement in reliability with R-curve behavior has already been discussed by Kendall et al.[10] Our analysis here confirms their qualitative discussion and provides in addition a parameter L/u_c which can be used as a guideline to the design of reliable sfrc.

The value of L/u_c is affected by material parameters of the matrix, fiber, and interface as well as the volume fraction, length, and radius of the fiber. A low value of K_c is desirable for reliability as discussed above. High values of interfacial friction, τ, fiber length, l, and small fiber radius, r, implies higher load carrying capacity for the fibers. Hence, the contribution of fiber bridging is increased and L/u_c decreases. A low value of E_f and high value of E_m is desirable as this will increase u_{cs}, the crack opening beyond which the bridging stress for a certain fiber is going to drop. One can notice from Fig. 2 that the area under the p vs u curve (which is an indication of the total work done in debonding and pulling out a fiber) increases with u_{cs}. A higher u_{cs} thus implies more work done on debonding and pulling out the fibers and hence a higher contribution of fiber bridging to the overall composite toughness. Increasing V_f implies higher load carrying capacity of the fibers but also implies an increase in K_c (through the increase in E_c as $K_c = (E_c/E_m)K_m$) and a decrease in u_{cs} for each single fiber. Thus, if other parameters are fixed, there will be an optimal value V_f for minimum L/u_c. By differentiating the expression for L/u_c (eq. 10) with respect to V_f, it is found that L/u_c is minimum when:

$$V_f = 1/[1 + \gamma^{1/2}(E_f/E_m)] \tag{11}$$

where γ is a correction factor for fiber length and orientation when calculating E_c. For all the numerical calculations in this paper, γ is taken to be 0.375.

An increase in τ will decrease L/u_c and hence improve reliability. However, τ cannot be increased without limit. If τ is too high, no interfacial debonding will be allowed and the crack, instead of circumventing fibers and leaving them behind as bridges, may just cut through the fibers. The composite reliability will then be very low. In such a case, the composite will also have a low toughness. Also, low values of K_c and E_c are desirable for high material reliability but will also lead to low composite strength and stiffness. In the design of composites, strength, reliability, stiffness, and toughness are all important mechanical properties to be optimized. Thus, a combination of fiber, matrix, and interfacial properties that can produce the best compromise between the various properties should be employed.

From this analysis, it is found that the material reliability of sfrc increases with decreasing L/u_c. Especially if $L/u_c < 0.38$, very high material reliability can be achieved and there can be a high probability for multiple cracking to occur. Therefore, it would be desirable to design sfrc with L/u_c below 0.38. However, since the present analysis is approximate (due to the use of an approximate crack profile and assumptions made in computing the composite p vs u relation), the transition value of 0.38 should be used with care. If L/u_c is designed

Table II. Values of u_c/l, L/u_c, and c_m for Some Short Fiber-Reinforced Ceramic Composites

Material System	Steel Fiber/ Concrete	High Strength Graphite/ Borosilicate		High Modulus Graphite/ Borosilicate	SiC Whisker/ Mullite
E_m (GPa)	30	65		65	140
E_f (GPa)	210	250		380	550
V_f	1	30		10	20
τ(MPa)	1.5	10	30	10	100
K_m (MPam$^{1/2}$)	1	0.75		0.75	2.2
r (μm)	100	4		4	0.4
l/r	100	100	40	100	100
u_c/l	6.7×10^{-4}	1.5×10^{-3}	1.8×10^{-3}	1.6×10^{-3}	9.2x10-3
L/u_c	2.874	0.182	0.263	0.363	0.176
c_m (μm)	2.82×10^5	77.7	37.4	343	14.5

to be significantly less than 0.38, good reliability can be assured. However, if in the final design, due to processing constraints on the fiber aspect ratio or volume fraction, L/u_c is close to 0.38, then either a more detailed analysis have to be carried out or experiments have to be done to study the behavior of the composite.

The values of u_c/l, L/u_c, and c_m are computed for several composite systems and tabulated in Table II. The results show 1% steel fiber reinforced concrete ($l/r = 100$, $r = 100$ μm) to be a material with relatively low reliability and 30% high strength graphite-reinforced borosilicate glass ($l/r = 100$, $r = 4$ μm) to be a very reliable material. Also, for the high strength graphite/borosilicate composite, $c_m = 77.7$ μm, and multiple cracking will take place if the inherent flaw size is greater than about 20 μm. This is very likely in short graphite fiber composites where fiber ends that act as stress concentrators can lead to the formation of flaws several times the fiber size. An aspect ratio of 100 may not be easily achieved in practice, due to fiber breakage during mixing. If an aspect ratio of 40 is employed for the high strength graphite fiber, and the fiber surface is treated to increase the interfacial friction to 30 MPa, L/u_c will be increased to 0.263, which still gives high material reliability. Also, with $c_m = 37.4$ μm, the probability for multiple cracking to take place is very high. To look at another example, if 10% high modulus graphite ($l/r = 100$, $r = 4$ μm) is used to reinforce borosilicate glass, L/u_c will be 0.363, which is very close to the transition value of 0.38. In this case, experimentation or a more detailed analysis is necessary to confirm the behavior of the composite. A final example is a 20% SiC whisker-reinforced mullite, with $\tau = 100$ MPa as estimated by Becher et al.[11] and $r = 0.4$ μm. This material can have very high reliability $L/u_c = 0.176$) if an aspect ratio of 100 is used. However, the current composite processing techniques usually limits the aspect ratio of whiskers to below 60, since L/u_c is inversely proportional to l^2. An aspect ratio of 50 will increase L/u_c to above 0.704, which is well above 0.38 and will lead to relatively low material reliability.

Limitations of the Current Study

The use of an approximate crack profile clearly imposes some limitations on the results of this study. While using the exact and approximate crack profiles should give rise to the same qualitative conclusions (as shown by Marshall et al.[2] for cfrc), the L/u_c value for transition between high L/u_c behavior and low L/u_c behavior may be different. A numerical analysis, similar to that employed in Refs. 2 and 3 can be carried out to obtain the exact L/u_c values.

Several other limitations arise from the assumptions made in deriving the composite p vs u relation. The p vs u relation for a single fiber is derived for the case of a purely frictional interface, and is only applicable to cases where interfacial toughness does not play an important part in the fiber debonding process. In more general cases, a more complicated p vs u relation that includes the contribution from interfacial toughness should be employed. In deriving the fiber bridging stress vs crack opening relation for the composite, three additional assumptions have been made. The assumption of uniformly distributed shorter embedded length is equivalent to assuming a uniform distribution of fiber in the matrix, and is usually justified in cases where powder and matrix are first mixed thoroughly together and then pressed. The assumptions of similar behavior of inclined and aligned fibers as well as zero fiber breakage are not satisfied in general and impose the strongest limitations on the results presented in this paper. Inclined fibers generally behave differently from aligned fibers and as the crack is opened up, inclined fibers are being bent and may break even if very short fibers are employed. Thus, with the assumptions made above, the results presented in this paper are only strictly valid for the cases where all the fibers are aligned with the loading direction. When the fibers are flexible enough to bend across the crack without breaking and the matrix is weak enough to either crack or yield at the fiber exit point (i.e., the point where the fibers exit into the crack) to reduce the direct stress acting on the fiber from the matrix, the present results may still be a good approximation. However, the case of flexible fiber and weak matrix is not usually found in ceramic composites. Therefore, for ceramic composites, the study of the effect of fiber inclination on crack-bridging and fiber breaking mechanisms is very important. Such a study is currently being conducted. It is expected that the result will lead to the derivation of a fiber-bridging stress vs crack opening relation generally applicable to random fiber composites. The first-cracking strength as a function of flaw size can then be obtained by the same methodology described in this paper.

Conclusion

The design of composites involves optimization of a number of properties. Strength, toughness, reliability, deformation capacity (pseudo-ductility), and environmental durability are some of the issues to be considered in composite design. This study identified L/u_c to be a parameter governing the behavior of sfrc and suggested a low L/u_c value to be a criterion for sfrc to have high reliability and pseudo-ductility. While further improvements in the theory is still necessary, the design criterion can be employed in practice provided the limitations are well understood.

References

[1] A. G. Evans, "The New High Toughness Ceramics," Report 1987-1, Univ. of Calif. Santa Barbara (1987).
[2] D. B. Marshall, B. N. Cox, and A. G. Evans, "The Mechanics of Matrix Cracking in Brittle-Matrix Fiber Composites," *Acta Metall.*, 3, 11, 2013–21 (1985).
[3] D. B. Marshall and B. N. Cox, "Tensile Fracture of Brittle Matrix Composites: Influence of Fiber Strength," *Acta Metall.*, 35, 11, 2607–2619 (1987).
[4] J. Aveston, G. A. Cooper, and A. Kelly, "Single and Multiple Fracture," in The Properties of Fiber Composites, NPL Conference Proceedings, IPC Science & Technology Publisher, 15–26 (1971).
[5] D. B. Marshall and A. G. Evans, "Failure Mechanisms in Ceramic-Fiber/Ceramic-Matrix Composites," *J. Am. Cer. Soc.*, 68, 5, 225–231 (1985).
[6] L. R. F. Rose, "Toughening due to Crack-Front Interaction with a Second-Phase Dispersion," *Mechanics of Materials*, 6, 11–15 (1987).
[7] N. Fares, "Crack Fronts Trapped by Arrays of Obstacles: Numerical Solutions Based on Surface Integral Representation," Accepted for Publication in ASME J. Appl. Mech. (1989).
[8] B. R. Lawn, Fracture of Brittle Solids, Cambridge University Press (1975).
[9] I. N. Sneddon and M. Lowengrub, Crack Problems in the Classical Theory of Elasticity, Wiley, New York (1969).
[10] K. Kendall, N.McN. Alford, and J. D. Birchall, "Weibull Modulus of Toughened Ceramics," in Advanced Structural Ceramics, MRS Symposium Proceeding, 78, 189–197 (1987).
[11] P. F. Becher, T. N. Tiegs, J. C. Ogle, and W. H. Warwick, "Toughening of Ceramics by Whisker Reinforcement," in Fracture Mechanics of Ceramics, Vol. 7, Plenum Press, 61–73 (1986).

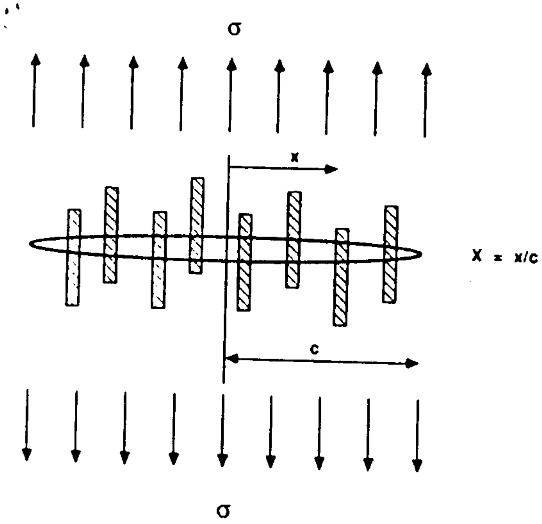

Fig. 1. A crack bridged by fibers under uniform tension.

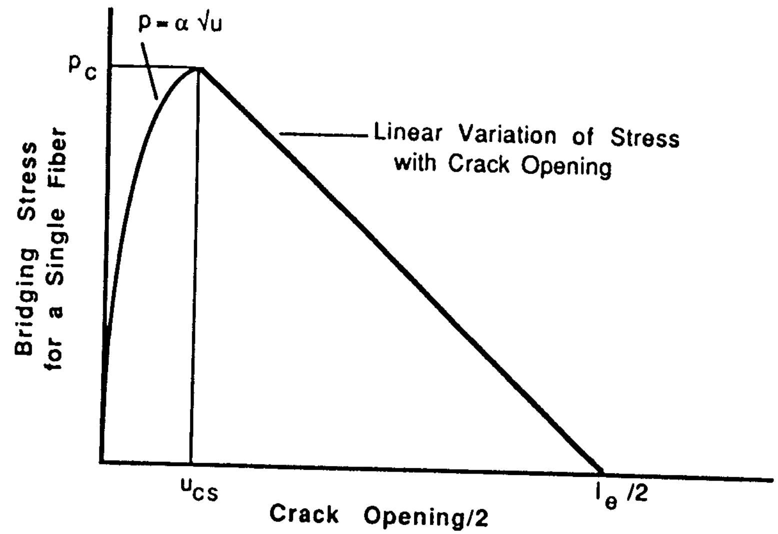

Fig. 2. Bridging stress vs crack opening relation for a single fiber.

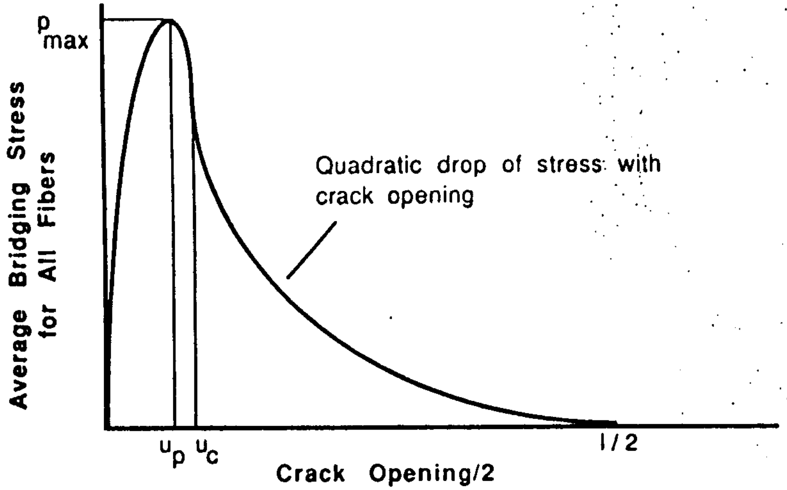

Fig. 3. A typical average bridging stress vs crack opening relation for a short fiber-reinforced composite.

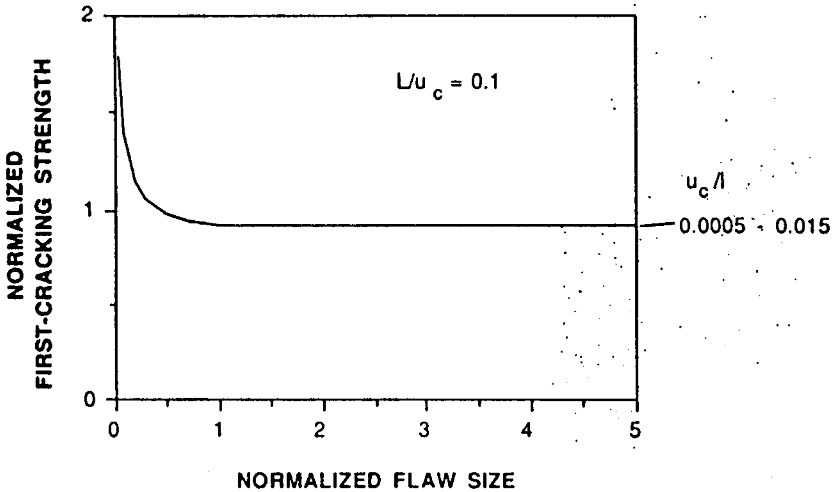

Fig. 4. Normalized first-cracking strength vs normalized flaw size for various values of u_c/l when $L/u_c = 0.1$.

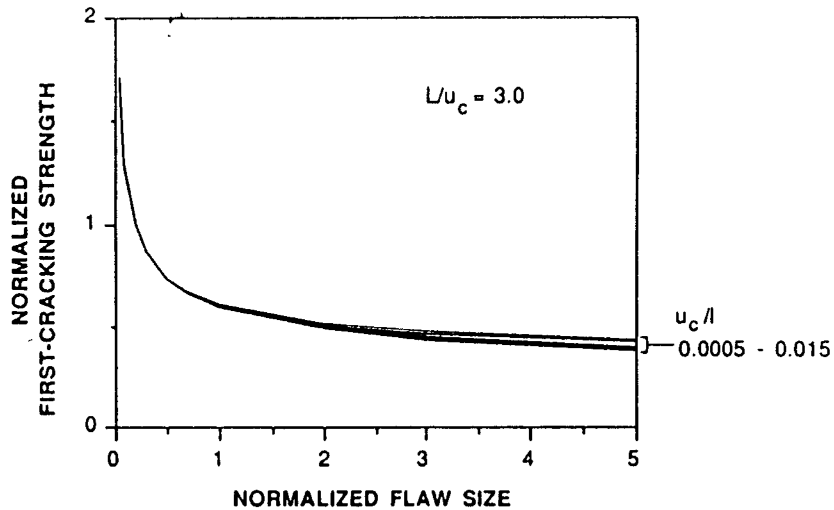

Fig. 5. Normalized first-cracking strength vs normalized flaw size for various values of u_c/l when $L/u_c = 3.0$.

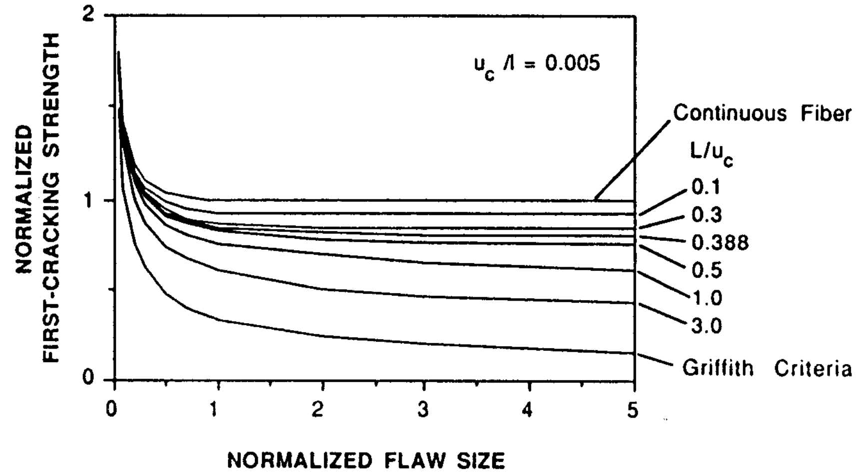

Fig. 6. Normalized first-cracking strength vs normalized flaw size for various values of L/u_c when $u_c/l = 0.005$.

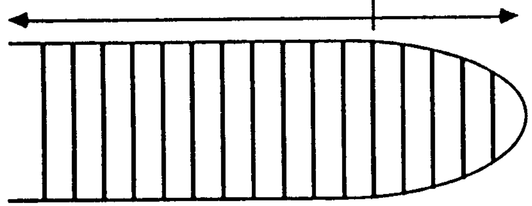

Fig. 7. Steady state cracking in a fiber composite.

Ceram. Eng. Sci. Proc. 10[9–10] pp. 1179–1190 (1989)

Residual Stresses and Damage in Unidirectional Model Composites

A. Chatterjee

AdTech Systems Research Inc.
Dayton, OH

Capt. J. W. Moschler

Air Force Inst. of Technology
WPAFB, OH

R. J. Kerans and N. J. Pagano

Air Force Materials Laboratory
WPAFB, OH

S. Mall

AFIT
WPAFB, OH

Unidirectional model composites were fabricated with silicon carbide fibers and different borosilicate glasses to study the effect of residual stress states on the damage progression in these composites. A specially designed straining stage was employed to study the failure modes in these materials under stepwise loading. Although both fiber and matrix cracks were observed in all specimens, the mechanisms of failure were found to be different and strongly dependent on the residual stress state in these materials.

Introduction

Due to the increasing demand for materials to withstand elevated temperatures, there is a growing effort in the development of ceramic composites. However without a basic relationship between the properties of constituents and interfaces, the understanding of the fundamental response characteristics of these materials cannot be optimized. Successful development of these composites therefore strongly depends on the development of such an understanding.