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# Development of Flexural Composite Properties and Dry Shrinkage Behavior of High-Performance Fiber Reinforced Cementitious Composites at Early Ages

by Yun Mook Lim, Hwai-Chung Wu, and Victor C. Li

*The results of an experimental study on the early age development of composite properties under flexural loading condition and on the dry shrinkage behavior under free or restrained boundary condition of High-performance fiber reinforced cementitious composites (HPFRCCs) are presented. Dry shrinkage of HPFRCCs with high cement content is found to strongly influence the early age composite properties—first cracking strength, first cracking strain, and elastic modulus. This phenomenon can be correlated with shrinkage cracking caused by a humidity gradient across the thickness of specimens. On the other hand, modulus of rupture and flexural toughness of HPFRCCs are not significantly affected by dry shrinkage. A much larger dry shrinkage of HPFRCCs, compared with ordinary concrete, was measured; nevertheless, a significant reduction of the maximum crack width is found from the restrained shrinkage test. Hence, water permeability of these HPFRCCs is expected to be sharply reduced compared with concrete, even though they have higher dry shrinkage.*

**Keywords:** dry shrinkage; modulus of rupture; residual stress.

## INTRODUCTION

Time-dependent behavior of cement-based materials such as concrete has been recognized as an important aspect of mix design. Typically, the material properties and the shrinkage of cement-based materials exhibit complex time-dependent behavior.[1] Thus, such time-dependent behavior is considered essential design factors for the safety and serviceability of civil engineering structures, especially during construction and at early age. In the construction industry, the early age properties of cement-based materials have become increasingly important because rapidly cured materials can accelerate a construction process or shorten a production cycle of precast members.[2]

The early age material properties of cement-based materials, as influenced by dry shrinkage, have been investigated theoretically and experimentally.[3] In some cases, the extent of dry shrinkage can significantly change the early age material properties and their development over time. For example, the flexural strength or modulus of rupture (MOR) of concrete specimens with no shrinkage cracks was found to be more than twice that with dry shrinkage cracks for certain specimen size and drying condition.[4] These studies have been performed using ordinary concrete.

Recently, many cement-based materials have been developed for potential civil engineering applications. High-Performance Fiber Reinforced Cementitious Composite (HPFRCC) is one of the most promising materials.[5,6] HPFRCCs have been engineered to satisfy various field performance requirements such as high durability, or impact load resistance. They have been shown to have outstanding combinations of strength and ductility or energy absorption capacity while achieving substantial pseudo-strain hardening (multiple cracking) behavior.[5] However, the early age properties of these HPFRCCs have not been carefully investigated.

In many cases, the compositions of these HPFRCCs are significantly different from those of ordinary concrete. Furthermore, pseudo-strain hardening behavior of HPFRCCs can only be achieved when certain micromechanical conditions are met.[6,7] Elastic modulus, fracture toughness of matrix, fiber/matrix interface bond strength, and initial flaw size are some of the microparameters that govern the composite performance, and many of these parameters are time dependent. Thus, the early age composite property development, the dry shrinkage, and its effect on composite properties of HPFRCCs cannot be simply extrapolated from the results of ordinary concrete.

## RESEARCH SIGNIFICANCE

In this study, the composite properties—first crack strength, first crack strain, elastic modulus, and modulus of rupture (MOR)—under flexural loading condition were measured at different ages (up to 30 days) along with the shrinkage response (up to 50 days) of the same material. These experimental results provide useful information to understand the early age composite property development of HPFRCCs. Additionally, these results form a valuable database in the design of HPFRCCs as a practical construction material.

## EXPERIMENTAL PROGRAMS

For the purpose of this study, two types of shrinkage test and flexural test were performed. Free shrinkage and restrained shrinkage were measured, and a four-point bend test was performed to evaluate age development of the composite properties. The four-point bend test was selected because of convenience of set-up and test. First crack strain, first crack strength, and ultimate flexural strength of the specimens were obtained from the flexural tests. Materials of specimens and testing details are described in the following sections.

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## Materials

In this study, randomly oriented, short-fiber composites showing multiple cracking behavior under flexural loading were used as a HPFRCC. The material compositions of these composites are reported in Table 1. Spectra (high-elastic modulus polyethylene) fiber with a 28-μm diameter was used. The only difference between composite C1 and C2 is fiber length. C1 composite (with 12.7-mm fiber length) contained fibers twice as long fibers as the fibers in C2 composite (with 6.35-mm fiber length). The workability of C1 was much worse than that of C2 by visual inspection, and this was the result of the higher fiber aspect ratio. The workability of C2 was not much different compared with that of C3. C3 is a plain mortar with identical composition to C1 and C2, with the exception that it contains no fiber.

Silica sand (50 to 70 size) was used in this study. No coarse aggregates were used. This was dictated by the need to maintain a low toughness matrix for satisfaction of the strain-hardening condition.[7,8] The sand/cement ratio of 0.5 was relatively low compared with ordinary concrete, which typically contains aggregate with aggregate-to-cement ratio of 4 to 5. For the comparison of shrinkage behavior, normal concrete with coarse aggregates (maximum size = 9.5 mm, cement: sand: aggregate = 1: 1.72: 1.72, and water-cement ratio = 0.45) was used in this study.

## Shrinkage specimens

The specimen configuration of the restrained shrinkage test is illustrated in Fig. 1, and the free shrinkage specimen has a 25.4 x 25.4 mm cross section and is 254 mm in length. One restrained shrinkage specimen and four free shrinkage specimens were cast for each composite using the same batch of material to minimize any batch variation. They were demolded approximately 24 hr after casting and cured under 100 percent relative humidity condition for 2 days after demolding. The specimens were then stored in an environmental chamber with controlled temperature and relative humidity (24 ± 2 C and 50 ± 5 percent) according to the standard test conditions, ASTM C 596-89 and ASTM C 157.

The free shrinkage and the crack width in each restrained shrinkage specimens were separately monitored up to 50 days after casting. The length change of the free shrinkage specimens was measured using a comparator with 0.00254-mm accuracy. In the restrained shrinkage specimen, the crack width was measured using a microscope with 100x magnification. Three points (12.7, 76.2, and 139.7 mm from the top of the specimens; see Fig. 1) were measured along the crack line for

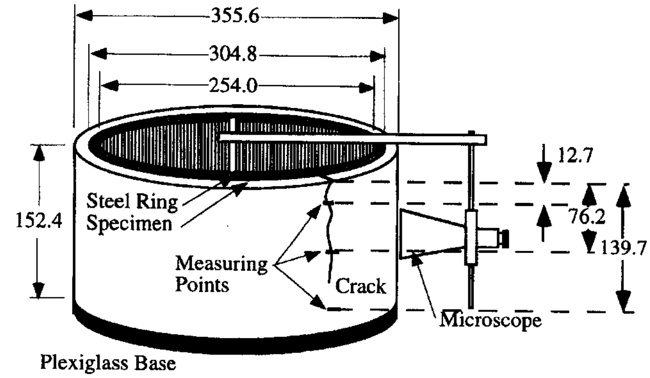



*Fig. 1—Specimen and measuring device for restrained shrinkage test (units in mm).*

### Table 1—Composition of composites (in weight)

| Material | Cement | Sand* | Water | Fiber† ($V_f$) | SF | SP |
|---|---|---|---|---|---|---|
| C1 and C2 | 1.0 | 0.5 | 0.35 | 0.02 | 0.1 | 0.01 |
| C3 | 1.0 | 0.5 | 0.35 | — | 0.1 | 0.01 |

*50 to 70 silica sand.
†Spectra 1000 in volume fraction.
SF = silica fume; SP = superplasticizer.

each crack, and the average value of these three measurements was reported as the crack width of each crack.

## Flexural specimens

The experimental configuration is illustrated in Fig. 2. The flexural specimens have a 38.1 x 38.1-mm cross section and are 152.4 mm in length. The span length of loading was 114.3 mm with a 38.1 mm center span length. A ball bearing was used to avoid uneven load distribution on the specimens. Fifteen flexural specimens for each composite type were cast together with the shrinkage specimens from the same batch of material mix.

The specimens were demolded after 24 hr, and three specimens were tested right after demolding in the first-day flexural test. Other specimens followed the same curing condition for the shrinkage specimens as described above, and were tested at 4, 7, 14, and 30 days after casting. A close-loop machine was used for the flexural tests. A concrete strain gage 25.4-mm long was attached on the tensile surface at the center of the specimens (Fig. 2) to measure the first crack strain of the composites. There was no difficulty in attaching strain gages on the specimens for wet condition tests because the surface was dry enough right after exposing to air (the total exposing time to air was less than 10 min before loading). The first crack strength and MOR were computed from the measurement recorded in the load cell of the loading machine. The elastic modulus was evaluated from the linear portion of the stress-strain curve. The detail of measuring for the first crack strain, the first crack strength, and the elastic modulus is in Reference 9.

Three specimens of C1 and C2 composites were tested for each age (a total of 15 specimens for each composite), and two specimens of C3 were tested for each age (a total of 10 specimens). In addition, nine specimens of C2 material were tested under a wet condition to investigate the effect of drying on composite properties. The wet conditioned specimens were tested at 4, 7, and 30 days after casting, and were performed within 30 min after removal from the curing tank. Apart from

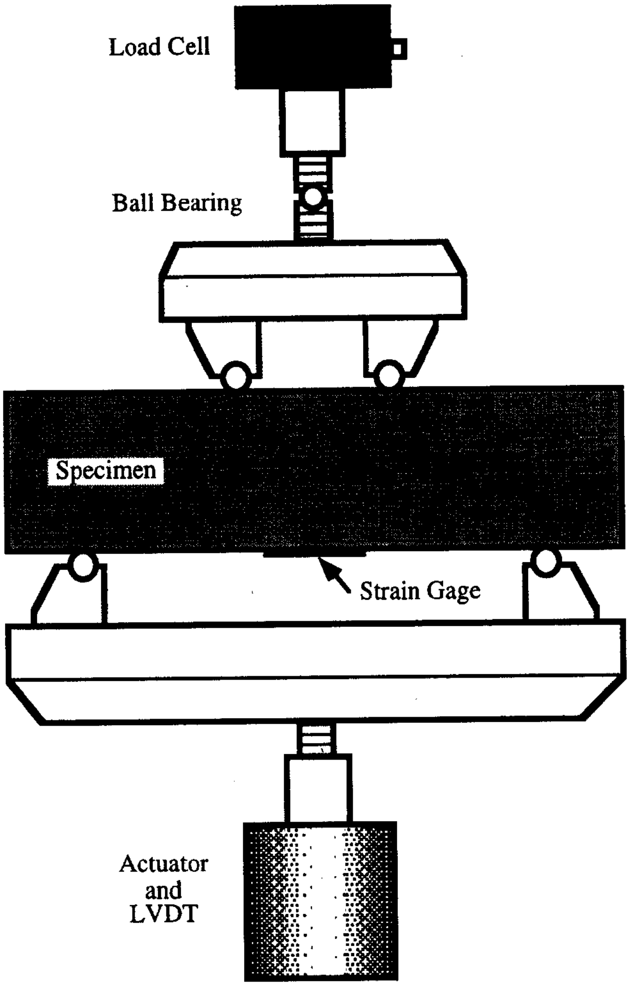



*Fig. 2—Testing setup for flexural property measurement.*

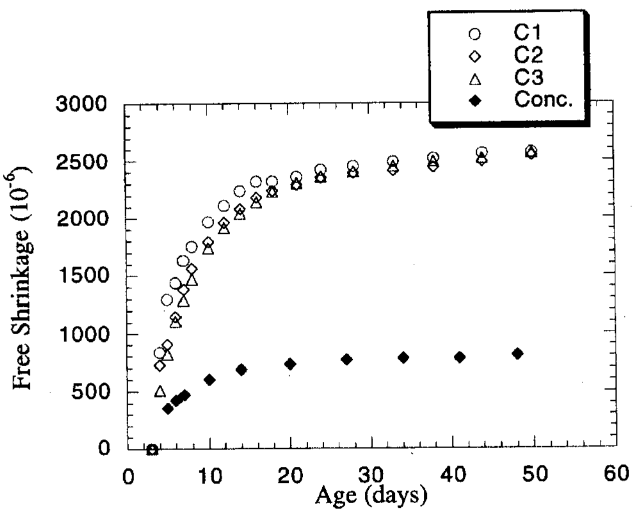



*Fig. 3—Free shrinkage development in HPFRCCs and concrete.*

normal curing conditions of C3 specimens, two additional specimens of C3 material were tested after a 29-day water curing. These two specimens were dried 24 hr prior to testing (a total of 30 day-old specimens). The average values of test results with error bars indicating maximum and minimum test results are presented below.

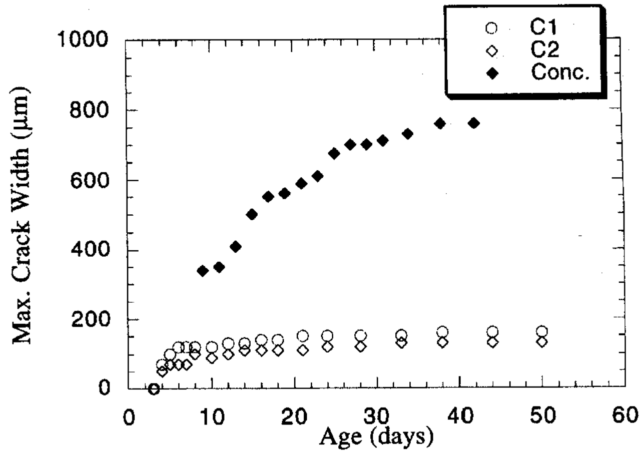



*Fig 4—Maximum crack width development of HPFRCCs and concrete.*

**Table 2—Weight increment of wet specimens**

| | C1 | C2 | C3 |
|---|---|---|---|
| Dry, gm | 1560.6 | 1591.0 | 1595.0 |
| Wet, gm | 1619.1 | 1636.7 | 1641.1 |
| Increment, percent | 3.75 | 2.87 | 2.89 |

## EXPERIMENTAL RESULTS

### Shrinkage behavior

The free shrinkage of HPFRCCs and plain mortar are illustrated in Fig. 3, together with the free shrinkage data of concrete.[9] Approximately three to four times larger shrinkage compared to plain concrete was measured, probably due to the high cement content and the absence of coarse aggregate in HPFRCCs. In the early stage (< 10 days), the free shrinkage of C1 was approximately 10 to 60 percent higher than that of C3, with C2 somewhere in between. However, the differences between the three materials diminished over time. C1 always showed higher free shrinkage compared with C2 and C3. The higher shrinkage in C1 compared with that in C2 might be attributed to a higher porosity in the composite due to a low workability. Table 2 contains the total weight of dry specimens (stored in the environmental chamber for 50 days) and wet specimens (soaked in water for 30 hr after 50 days of shrinkage testing). The increment of the weight can indirectly correlate with the amount of pore in the tested specimens.

Fig. 4 illustrates the maximum crack width of HPFRCCs and a normal concrete. The C3 cracked within 24 hr after casting, and the restrained specimen had already a 1100-μm crack opening on the first day. The crack width of C3 reached 11,000 μm at 20 days and remained the same thereafter. C1 and C2 had 23 and 19 cracks, respectively, in the restrained specimens, and the maximum crack width was about 1/5 that of concrete. The average crack widths were 62.45 and 72.81 μm at 50 days in C1 and C2, respectively. Although the total crack widths (sum of averaged width of all cracks) of C1 and C2 were approximately two times higher than that of concrete, the water flow rate might be dramatically reduced as a result of a significant reduction in maximum crack width. The flow rate was found to scale with the third power of crack width in Tsukamoto's study.[10] The flow rate of C1 or C2 could be about 100 times less than that of concrete under the assumption that water flow rate only increases linearly with the number of cracks. Reduction of the

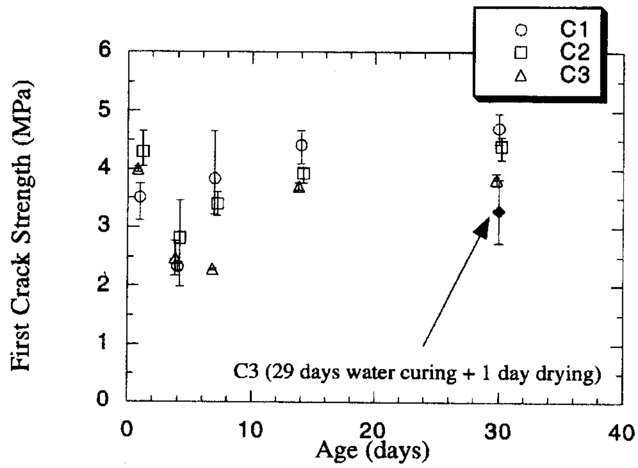



*Fig. 5—First crack strength development.*

water flow rate can decrease corrosion of reinforcing bars in concrete structures. Thus, a drastically reduced crack width resulting in low water permeability can have a significant positive effect on long-term durability of concrete structures.

## Flexural behavior

Fig. 5 shows the development of first crack strength with time. The first crack strengths were calculated from the measured loads when the strain reading from the attached strain gage suddenly changed in response to the first cracking. Whenever a first crack occurred on the tensile surface, the strain gage detected a physical change of materials. The first cracking strengths at 4 days were found significantly lower than that at 1 day in C1, C2, and C3 specimens that experienced drying.

This phenomenon of strength reduction can be attributed to shrinkage cracking caused by material internal restraint. Drying is usually not uniform through the thickness of specimens or structural members [see Fig. 6(a)], and this uneven humidity change leading to a shrinkage strain gradient [Fig. 6(b)] provides an internal restraint.[11] An internal stress is then developed as a result [Fig. 6(c)]. The degree of shrinkage strain and stress in different specimen sizes was calculated.[12] The shrinkage strain at the surface of the specimens was determined to be in the range of 0.0004 to 0.0011. The stress due to this shrinkage strain was 2 to 7 times higher than the tensile strength of materials when the effective elastic modulus and the tensile strength were assumed 20.0 GPa and 3.0 MPa, respectively.[12] Such high stress could readily develop surface cracks. In this current study, surface cracks were observed prior to testing in all our flexural specimens except the first day specimens (tested right after demolding) and the wet-conditioned specimens. The development of such surface cracks at early ages might therefore affect the first crack strength leading to a decrease at the 4 days testing.

To confirm the above hypothesis, additional C2 specimens were tested without undergoing the drying process. In Fig. 7, C2-W means the C2 specimen was under wet conditions up to testing. The first crack strengths of the wet-conditioned specimens were significantly improved as much as two times at later age. In the wet condition test, there was no chance to develop dry shrinkage cracks on the specimen surface; hence, first crack strength and strain could be much improved.

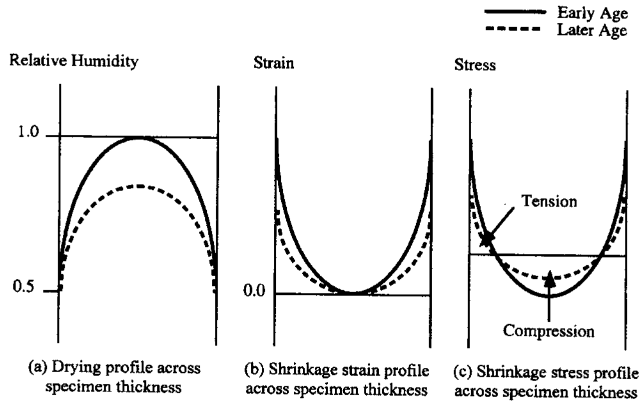



*Fig. 6—Drying, shrinkage strain, and shrinkage stress profiles.*

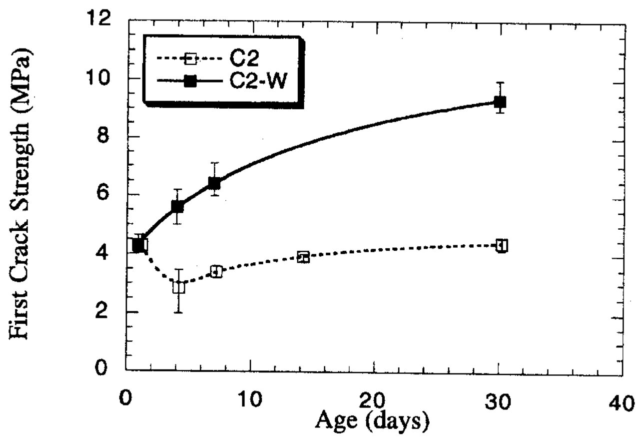



*Fig. 7—First crack strength development in C2 with dry and wet (C2-W) conditions.*

In Fig. 5 the first crack strengths show an increasing trend after the drop at 4 days. This may be explained by two effects. One is the hydration process under 50 percent relative humidity [actual humidity inside specimens > 50 percent; see Fig. 6(a)]. The matrix toughness $(K_m)$ increases with this continuing hydration process.[13] The other is a reducing residual stress due to shrinkage at the crack tip. The amount of residual stress is reduced at later age [illustrated in Fig. 6(c)]. Also, the relaxation due to creep might affect the reduction of residual stress.

C1 had the lowest strength at 1 day and 4 days among the tested materials. This was probably because of the presence of larger voids due to poor workability, and a larger amount of free shrinkage at an early age (Fig. 3). However, after 7 days, C1 showed higher strengths than C2. The possible reason for this phenomenon is that the residual stress in C1 might be less than that of C2. At the same age, the drying free shrinkage of C1 was always higher than that of C2 (Fig. 3). This means that the rate of drying in C1 could be faster, and the humidity profile at the same age might be flatter [equivalent to a dotted line for C1 and a solid line for C2, shown in Fig. 6(a)]. Thus, the residual stress of C1 might be reduced, leading to higher first crack strengths at later age.

Two additional C3 (plain mortar) specimens were tested under a different curing condition (29 days of water curing and 1 day of drying). Higher flexural strength than that of the other mortar specimen associated with a continuing hydration under water was expected, but comparable flexural strength was actually measured. This is probably attributed to shrinkage surface cracks that occurred during the 1-day drying period prior

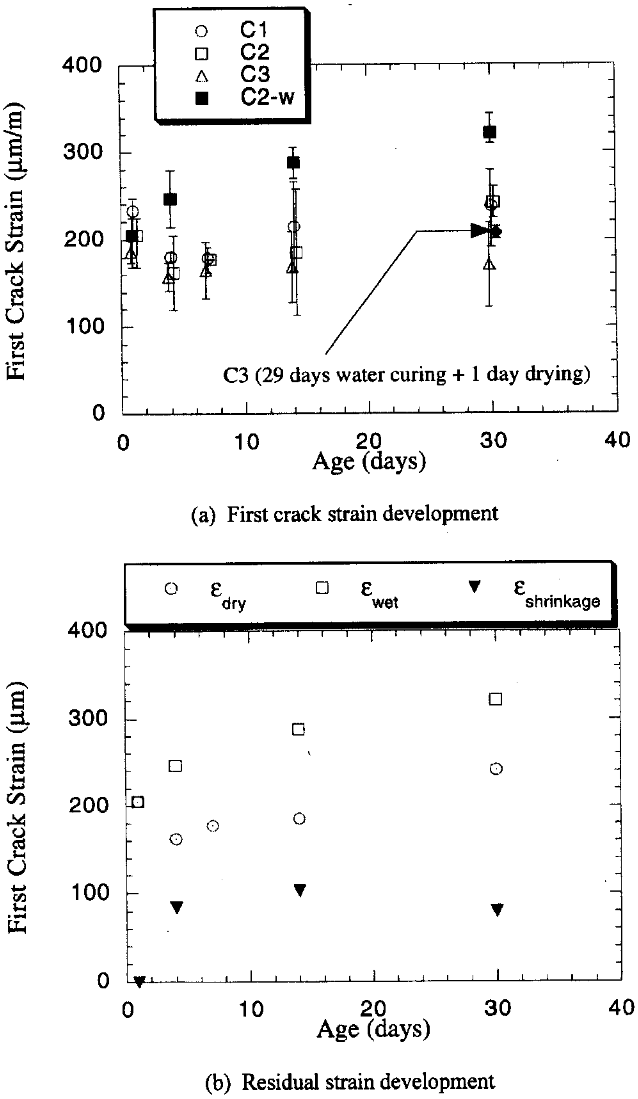



*Fig. 8—First crack strain and residual strain development.*

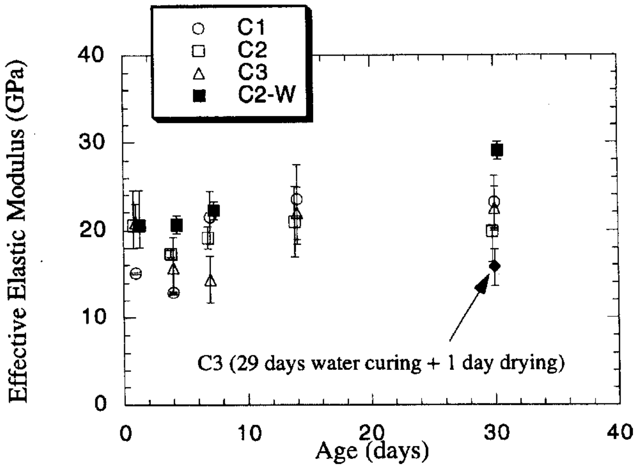



*Fig. 9—Effective elastic modulus development.*

to testing. The surface cracks due to dry shrinkage might affect the flexural strength more than the improvement due to the continuing hydration under water. The continuing hydration effect can be seen in increasing the first crack strength in the C2-W case since the specimens were tested immediately after removal from the curing bath so that the C2-W specimens had no chance to generate dry shrinkage cracks.

Fig. 8(a) illustrates the first crack strain capacity development with time. The strain data were recorded from the strain gages. The first crack strain dropped at 4 days and then increased almost linearly with time. Fig. 8(a) also includes the first crack strain of specimens without undergoing drying process (C2-W). There was no strain reduction at early age, and the strain capacity at 30 days was 50 percent more than at 1 day. This is consistent with the observed trend of the first crack strength. The total strain experienced by the material on the tensile side of the specimens was equal to the sum of the tensile strain due to external loads and the residual tensile strain due to differential dry shrinkage. For the first-day specimens, the residual strain was not yet developed or negligibly small. The residual strain then developed with age, but dropped with later age [see Fig. 6(b)] as moisture equilibrium was established. This residual strain development with time due to dry shrinkage is shown in Fig. 8(b). The difference between $e_{wet}$ (strain capacity of C2-W) and $e_{dry}$ (strain capacity of C2) is illustrated as $e_{shrinkage}$ (residual strain due to dry shrinkage).

The effective elastic modulus was determined from the slope of the measured stress versus strain curves. The measured stress/strain curves had a linear relationship up to the first crack point; thus, the slope was calculated from the first crack strength divided by the first crack strain. This elastic modulus is defined as an effective modulus because the compliance of the system accounts for the presence of microcracks due to dry shrinkage. The effective elastic modulus development with time is shown in Fig. 9. A trend consistent with that of first crack strength/strain is observed. In ordinary cementitious composites, including concrete, the elastic modulus is reported to continuously increase with time. Elastic modulus is usually measured under compressive loading and is minimally affected by surface cracking. This probably explains why different moduli values were obtained from tensile test and compressive test[14] in a HPFRCC.

The variation of modulus of rupture (MOR) with time is reported in Fig. 10. The fiber composites showed multiple cracking under flexural loading in specimens of all ages in C1 and C2, but not in C2-W. C2-W and C3 (plain mortar) respectively showed quasi brittle and brittle failure with only one macrocrack. A noticeable MOR drop at early age was only observed in C3, whereas MORs remained almost constant in C1 and C2. This is consistent with our expectation (based on numerical calculation of the bridged crack beam model) that MOR of the fiber composites with stable matrix crack growth[15] is mainly influenced by fiber bridging and not affected by initial defect size in the composite, especially if the crack-like defects are fully bridged by fibers. In our case, the shrinkage cracks are expected to be bridged by fibers. Further support can be found from the insignificant difference of MOR between C2 and C2-W. The only difference between these two was the initial defect size due to drying procedure. However, they showed different cracking behaviors—multiple cracks (3 to 4 cracks) in C2 and a single crack in C2-W. The appearance of multiple cracks in C2 could be attributed to propagation of surface cracks during the ascending portion of the flexural loading. On the contrary, no surface shrinkage crack existed in C2-W; hence, only a single crack was observed.

A much different MOR between C1 and C2 composites was expected since the fiber length of C1 was twice that of C2.

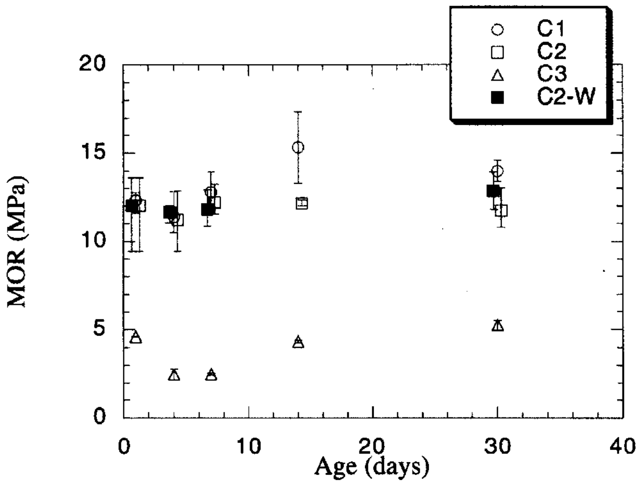
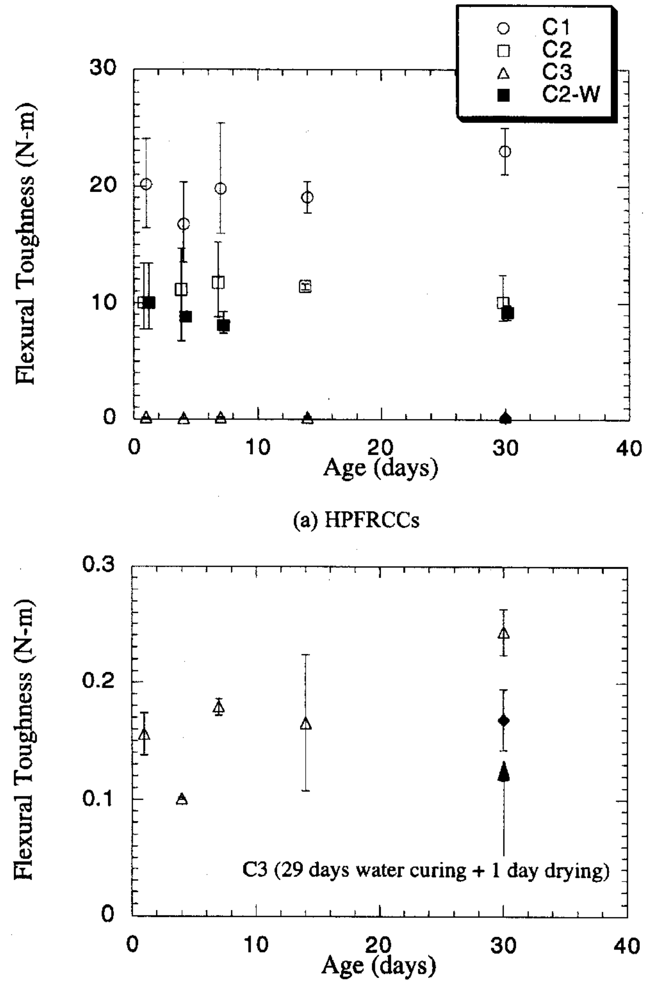



*Fig. 10—Modulus of rupture development.*




(a) HPFRCCs

(b) Plain matrix (expanded vertical scale)

*Fig. 11—Flexural toughness development with age.*

However, the MOR was not significantly different, especially at early age. This may be explained by a possibly different fiber/matrix interface bond strength in C1 and C2 composites, even if they had the same material composition with the exception of the fiber length. This possibly lower bond strength in C1 material may be a result of poor workability, which makes compaction more difficult in a composite with long fibers. The fast saturation of MOR (which occurred prior to the fourth day) with respect to age, in contrast to a much slower first crack strength development, was consistent with the recent findings of a fast saturation of bond strength with time (2 to 3 days in Spectra fiber case).[16]

Fig. 11 illustrates the flexural toughness [total area under the load-deflection curve from zero defection to the end of the tests (almost zero load)] development with time. The flexural toughness in C1 composites was about twice that of C2 composites due to longer fiber pull-out in C1. On the other hand, the flexural toughness in C2 was always higher than that of C2-W because of a larger deflection at the peak load. The large deflection of C2 mainly came from multiple cracking. Additionally, about 100 times improvement in flexural toughness was found between C1 and plain mortar C3.

## FURTHER DISCUSSION

The first crack strength and the effective elastic modulus of HPFRCCs under flexural loading were significantly affected by the drying process (C2 and C2-W in Fig. 7 and Fig. 9). The C2-W case showed the flexural properties of an ideally cured specimen without surface cracks as a result of dry shrinkage. However, the elimination of the drying process is impossible in real applications. Thus, these material properties can only be improved with proper curing to eliminate surface cracks or, with proper matrix design, to reduce the amount of dry shrinkage. Proper curing with humidity control may be applied to precast members such as cementitious composite wall panels. However, it may be impractical for cast-in-place structural members at ordinary construction sites. In this case, matrix design to reduce the amount of dry shrinkage is necessary. The usage of a shrinkage reducing agent (SRA) can be effective. Hence, the incorporation of SRA might improve the flexural first crack strength and strain of HPFRCCs (C2 curve approaching the C2-W curve in Fig. 7). More intensive research will be needed on this subject.

The restrained shrinkage specimen of C3 material (plain mortar) was cracked within 24 hr after casting, much before the onset of drying. Other shrinkage mechanisms, such as autogeneous shrinkage or thermal shrinkage, may dominate C3 material. Materials like C3 with a low water/cement ratio and silica fume, have been shown to exhibit very high autogeneous shrinkage in early age.[17] An autogeneous shrinkage strain about $4.0 \times 10^{-4}$ was reached only 20 hours after casting,[18] which is far beyond the first crack strain of plain mortar (less than $3.0 \times 10^{-4}$).

## CONCLUSION

The shrinkage response of HPFRCCs was measured in this study. The free shrinkage of HPFRCCs and plain mortar was higher than that of typical concrete, as expected. Nevertheless, the maximum crack widths in HPFRCCs were significantly reduced (about 1/5 of concrete), attributed to fiber bridging. Also, the addition of synthetic fiber (high-elastic modulus polyethylene fiber) had an insignificant effect on free shrinkage.

Development of composite properties under flexural loading condition is presented in this study. The first crack strength and strain were found to be significantly affected by surface microcracks caused by dry shrinkage. These surface microcracks probably resulted from humidity gradient across the specimens. Many microcracks were visually detected in all but the first day and the wet-conditioned specimens, which did not undergo the drying process.

The presented data might provide insight into understanding HPFRCCs and suggest design guidelines in real applications. Also, if the shrinkage effect of HPFRCCs on the first crack strength and the effective modulus could be minimized, such as by the addition of a shrinkage reducing agent, the development of such composite properties could be continuously improved without a drop at early age that is discovered in this study for the first time. Such improvement can accelerate the time of on-site construction or shorten the production cycle of precast members of HPFRCCs.

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## REFERENCES

1. Mindess, S., and Young, J. F., *Concrete*, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1981.

2. Han, N., and Walraven, J. C., "Creep and Shrinkage of High-Strength Concrete at Early and Normal Ages," *Advances in Concrete Technology*, SP-154, V. M. Malhotra, ed., American Concrete Institute, Farmington Hills, Mich., 1995, pp. 73-94.

3. Raphael, J. M., "Tensile Strength of Concrete," ACI JOURNAL, *Proceedings* V. 81, No. 2, Mar.-Apr. 1984, pp. 158-165.

4. Planas, J., and Elices, M., "Dry Shrinkage Eigenstresses and Structural Size Effect," *Fracture Mechanics of Concrete Structures*, Z. P. Bazant, ed., Elsevier Applied Science, London, 1992, pp. 939-950.

5. Naaman, A. E., and Reinhardt, H. W., "Characterization of High-Performance Fiber Reinforced Cement Composites—HPFRCC," *High Performance Fiber Reinforced Cement Composites*, V. 2: HPFRCC-95, Ann Arbor, Mich., 1995.

6. Li, V. C.; Mihashi, H.; Wu, H. C.; Alwan, J.; Brincker, R.; Horii, H.; Leung, C.; Stang, H.; and Maalej, M., "Micromechanical Models of Mechanical Response of HPFRCC," *High-Performance Fiber Reinforced Cement Composites*, V. 2: HPFRCC-95, A. E. Naaman and H. W. Reinhardt, eds., University of Michigan, Ann Arbor, Mich., 1995, pp. 43-95.

7. Li, V. C.; Mishra, D. K.; and Wu, H. C., "Matrix Design for Pseudo-Strain-Hardening Fiber Reinforced Cementitious Composites," *Materials and Structures*, V. 28, 1995, pp. 586-595.

8. Li, V. C., and Hashida, T., "Engineering Ductile Fracture in Brittle Matrix Composites," *Journal of Material Science Letters*, V. 12, 1993, pp. 898-901.

9. Lim, Y.-M. "Interface Fracture Behavior of Rehabilitated Concrete Infrastructures Using Engineered Cementitious Composites," PhD dissertation, Department of Civil and Environmental Engineering, University of Michigan, Ann Arbor, Mich., 1996.

10. Tsukamoto, T., "Tightness of Fiber Concrete," *Darmstadt Concrete*, V. 5, 1990, pp. 215-225.

11. Bazant, Z. P., "Mathematical Models for Creep and Shrinkage of Concrete," *Creep and Shrinkage in Concrete Structures*, Z. P. Bazant and F. H. Wittmann, eds., John Wiley & Sons, 1982, pp. 163-256.

12. Planas, J., and Elices, M., "Dry Shrinkage Effect on the Modulus of Rupture," *Creep and Shrinkage of Concrete*, Proceedings of the Fifth International RILEM Symposium, Z. P. Bazant and I. Carol, eds., Barcelona, Spain, E & FN Spon, 1993, pp. 357-380.

13. Zollinger, D. G.; Tang, T.; and Yoo, R. H., "Fracture Toughness of Concrete at Early Ages," *ACI Materials Journal*, V. 90, No. 5, Sept.-Oct. 1993, pp. 463-471.

14. Li, V. C., "Performance of ECC Materials in OHNO Shear Beams," UMCEE *Report* No. 92-13, Department of Civil and Environmental Engineering, University of Michigan, Ann Arbor, Mich., 1992.

15. Li, V. C.; Lin, Z.; and Matsumoto, T., "Influence of Fiber Bridging on Structural Size Effect," submitted to *International Journal of Solid and Structures*, 1996.

16. Chan, Y. W., and Li, V. C., "Age Effect on the Characteristics of Fiber/Cement Interface Bond," submitted to *Journal of Material Sciences*, 1995.

17. Stang, H., "Significance of Shrinkage Induced Clamping Pressure in Fiber-Matrix Bonding in Cementitious Composite Materials," *Advanced Cement-Based Materials*, V. 4, No. 3-4, 1996, pp. 106-115.

18. Dela, B., "Shrinkage Cracking in High-Strength Concrete," *Building Materials Laboratory and Department of Structural Engineering*, Technical University of Denmark, 1994.