



# Repair and retrofit with engineered cementitious composites

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## Abstract

This article presents the novel use of a super ductile fiber reinforced cementitious composite for repair and retrofit of concrete structures. Research in repair and retrofit demands immediate attention because of rapidly deteriorating and heightened safety requirements of civil infrastructures worldwide. The strain-hardening Engineered Cementitious Composites have been developed with the aid of fracture mechanics and micromechanics. It is emphasized that material ductility, and not just strength, can translate into strong and ductile structural performance. This article is extended from an original version presented as a Principal Lecture at the FRAMCOS-3 Conference at Gifu, Japan in October, 1998. © 2000 Elsevier Science Ltd. All rights reserved.

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## 1. Introduction

Worldwide, about 10% of GDP derives from infrastructure construction. In the US alone, infrastructure construction is a four hundred billion dollar industry involving six million jobs. Approximately \$17 trillion worth of infrastructures are in place. Infrastructures in many industrialized countries are aging. In the US, the interstate highway system is in disrepair. Almost 40% of US bridges are in some state of serious deterioration [1]. Simultaneously, it has become clear that many existing structures no longer meet today's safety standard. Examples abound in structures located in seismic regions, both in the US and in Japan. Obviously, total replacement of deteriorated structures or under-designed structures is economically infeasible. Instead the solution must lie in repair and retrofit

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technologies. The success or failure of repair and retrofit technologies often depends on the choice of materials.

Concrete as a structural material has undergone several important phases of development. In the early 1900 and around 1940s, steel reinforced concrete and prestressed concrete established themselves as viable alternatives to steel as major construction materials. Around 1970, high strength concrete became commercialized with the arrival of silica fume and superplasticizer as chemical additives. The development of high strength concrete continues to impact the design of taller, longer and bigger infrastructures. Fiber reinforced concrete (FRCs) began its broader acceptance by the practice community in the 1980s, although mostly limited to non-structural use. At the moment we are undergoing a phase of structural FRCs being applied in infrastructures in which the fibers are expected to carry loads.

High performance fiber reinforced cementitious composites can be designed systematically, based on the understanding of how crack-like defects in the composite grow and the influence of fiber bridging on the mode of composite failure. Fracture mechanics plays a significant role in the development of composite design strategy. The literature on the use of fracture mechanics for FRC or FRC structures is expanding (e.g. [2–5]). These represent complementary work to those conducted by Kendall et al. [6] on relating the microstructure to the strength of cement based materials. While reducing macro-defect size leads to higher strength, fiber bridging provides ductility to the composite. Cementitious composites with novel properties are emerging for structural applications.

The purpose of this article is to illustrate the potential enhancements of structural safety limit state and the durability of service limit states brought about by the use of an engineered cementitious composite. In so doing, the linkages of scales at the micro, meso and macro levels in materials and structures are emphasized. The contributions of fracture mechanics to high performance cementitious composite development should be evident.

## 2. ECC design

The design and properties of the engineered cementitious composites (ECCs) have been discussed at length [5]. Here, we provide a brief synopsis, focusing on those properties most relevant to the repair and retrofit example applications to follow. There are four performance targets for ECC: (1) high performance, (2) flexible processing, (3) short fibers at moderate volume fraction, and (4) isotropic properties. By high performance, we mean tensile ductility here, because this property appears to provide the greatest enhancement to infrastructural needs, and also appears to be the bottle neck property when viewed in light of the great strides made in high strength concrete in recent years. However, ductility is not exclusive to other desirable features such as durability, high strength or self-compacting rheological behavior. (In fact, a self-compacting ECC has recently been developed [7].) High ductility in the form of strain-hardening has been achieved with some FRCs which utilize large amount and/or continuous fibers (and therefore violates economic constraints), or requires highly specialized processes not easily implementable on a construction site. Apart from cast processing, ECC has also been extruded into pipe element [8]. The target of isotropic properties are especially useful when the stress field is multi-directional, or shifts with changing load conditions. Weak planes in the composite material should be avoided. Regular FRC satisfies some of these targets but do not in general possess high tensile ductility especially when measured in uniaxial tension or fracture toughness test.

Our goal is to design ECCs which meet all four targets. The underlying technique is to tailor the microstructure of the composite based on the understanding of the mechanical interactions between the fiber, interface and matrix phases in the composite under load. Fracture mechanics is prominently utilized at the meso level of cement matrix crack propagation behavior, and at the micro level in the

fracture debonding process of the fiber/matrix interface. For example, the concept of steady state cracking as a requirement for the condition for transition from single crack quasi-brittle failure mode to multiple crack ductile failure mode has been extensively analyzed [9–11], using the energy balance concept of steady state crack propagation. Micromechanics deals with the mechanical crack bridging action of the fibers, in the form of a stress-crack opening relationship. Specifically, the energetics of tunnel crack propagation along fiber/cement matrix is used to quantify the debonding process for those fiber/matrix systems characterized by chemical bond [12–14]. Statistics is introduced to describe the random nature of preexisting microcrack size, and the random location and orientation of fibers [15,16] in the composite. These analytic tools of fracture mechanics, micromechanics and statistics (often lumped together in the simplified terminology of ‘micromechanics’) together generate a theoretical model which can be inverted to serve as a composite tailoring guide. Such a guide is utilized in the design of ECCs.

To appreciate the power of this tailoring procedure, it is useful to recognize that each of the three composite phases fiber, matrix and interface has its own set of parameters. For example, the fiber is characterized in terms of its elastic modulus, tensile strength, length, diameter, and volume fraction. The matrix is characterized in terms of its toughness, elastic modulus and initial flaw size distribution. The interface, or more generally the fiber/matrix interaction parameters, include the friction and chemical bond properties and the snubbing coefficient [17]. For some fibers, strength reduction factors are also needed to describe the reduction of fiber strength when pulled at an inclined angle [18]. These parameters together govern the composite behavior. In particular, it would be desirable to determine the specific combination of these parameters which would give rise to composite strain-hardening as opposed to tension softening typical of regular FRCs. Because of the large set of parameters, empirical means to find the right combination is practically impossible. Micromechanics allows a systematic means to determine the transition from quasi-brittle behavior to ductile behavior, with the smallest amount of fiber. This amount, known as the critical fiber volume fraction  $V_f^{\text{crit}}$ , is strongly dependent on the matrix toughness, the interface bond property and the fiber aspect ratio. By tailoring these properties, it is possible to determine a  $V_f^{\text{crit}}$  small enough for regular processing and at the same time satisfy economic constraints. Simultaneously, high ductility is achieved.

To emphasize the importance of tailoring rather than forcing as much fiber into the composite as possible, Fig. 1 shows that an FRC with  $V_f = 7\%$  has quasi-brittle behavior in contrast to the strain-hardening behavior of an ECC with only  $V_f = 2\%$ , under uniaxial tensile loading. In this ECC the fiber aspect ratio is twice that of the FRC, while everything else remains identical. This example reveals that with the lower aspect ratio ( $L_f/d_f = 167$ ), the  $V_f^{\text{crit}}$  exceeds 7%, while for  $L_f/d_f = 334$ , the  $V_f^{\text{crit}}$  is lower than 2%. The ideally brittle behavior of a plain matrix specimen with no fiber is also shown.

Fig. 2 illustrates the influence of interfacial bond properties ( $G_d$ , an interfacial fracture toughness and  $\tau_i$ , interfacial friction) on  $V_f^{\text{crit}}$  for a composite containing fibers with chemical bond with the cement matrix. For a given  $\tau_i$ , high  $G_d$  raises the  $V_f^{\text{crit}}$  due to a greater propensity for fiber rupture. For a given  $G_d$ , the U-shaped curve means that there is a range of  $\tau_i$  which optimizes composite performance by creating strong fiber bridging during multiple cracking and yet adequate energy absorption by frictional sliding and limiting fiber rupture. More details of the analyzes of composite behavior with chemical bond interfaces can be found in Ref. [14].

### 3. ECC tensile and fracture related properties

In the following, we review the tensile and fracture properties of a ECC reinforced with high-modulus Polyethylene fibers (PE-ECC), which have 38  $\mu\text{m}$  of diameter, 117 GPa of elastic modulus, and 2400 MPa of tensile strength. Typical PE-ECC has been achieved with 1–2% of fiber volume fraction and

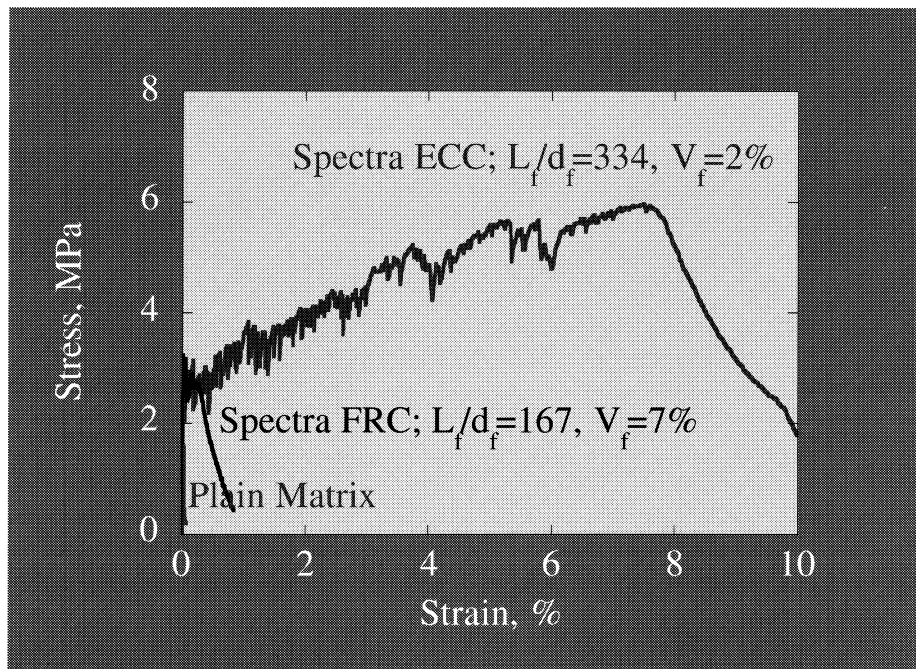


Fig. 1. Stress–strain curves demonstrating the importance of micromechanics based composite design.

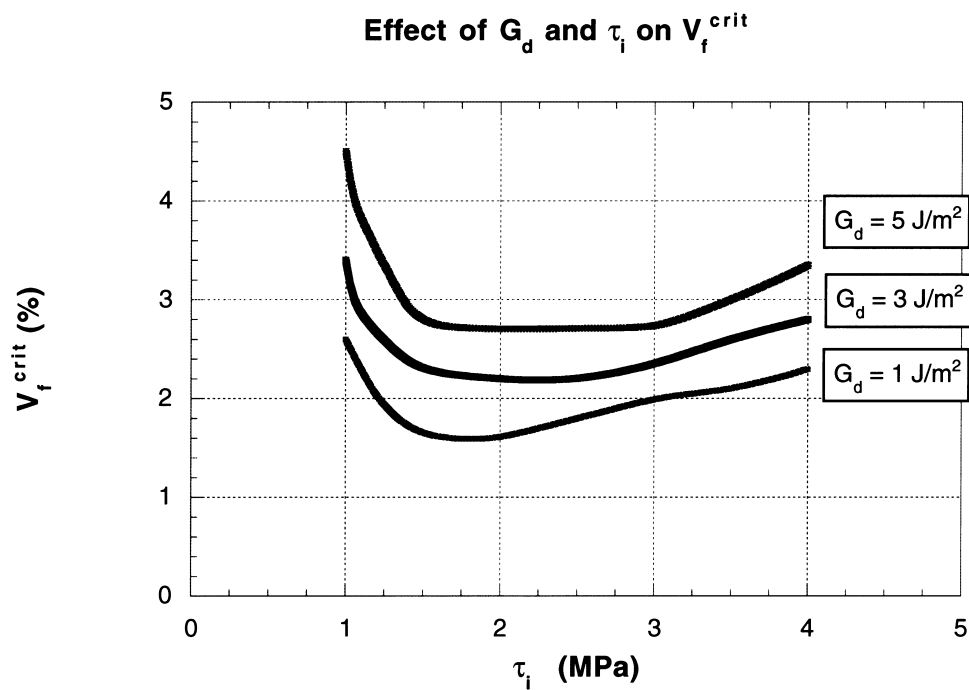


Fig. 2. Dependence of  $V_f^{crit}$  on the chemical bond  $G_d$  and interfacial friction  $\tau$ .

matrix mix proportion such as in Table 1. These properties are relevant to the application examples in repair and retrofit to be discussed in the following sections. Details of testing technique, material composition and other mechanical properties tested can be found in Ref. [1]. Other examples of ECC applications can be found in Ref. [19].

Fig. 3 shows the uniaxial tensile behavior of the ECC. The first crack strength (at the bent over point) can be adjusted by the cement or mortar matrix composition. As shown, it is lower than that of a typical steel FRC. Often times, a lower first crack strength can be desirable if damage initiation is needed at limited load magnitude such as in energy absorption devices. After first cracking, the all-important strain-hardening process begins, accompanied by inelastic deformation and load capacity increase. This continues until about 5.6% in tensile strain capacity for this example, when microcracking saturates and a localized fracture finally forms. Beyond this stage, tension softening as in the case of a regular FRC results.

The fracture behavior of an ECC compact tension specimen is shown in Fig. 4. Note the extensive damage development around the initial notch. The width of the inelastic zone is approximately 20 cm, resulting in a fracture toughness about 30 kJ/m<sup>2</sup> [20]. ECC is extremely damage tolerant. Fig. 5 shows the damage pattern of a double edge notched specimen loaded in tension. Diffusion of the microcrack damage away from the notches can be clearly observed. This strain redistribution renders the ECC notch insensitive, as can be seen in the failure load plot also shown in Fig. 5.

In summary, ECC is an extremely ductile cementitious composite designed with mechanical science. Tensile strain-hardening with strain capacity exceeding 2% can be achieved with fiber content less than 2% by volume. The high ductility and damage tolerance have important implications in structural performance, as will be described in the next section. The moderate amount of discontinuous fibers allow meeting cost and processing constraints and therefore is suitable for application in construction sites as well as in pre-cast plants.

## 4. Applications of ECC in repair and retrofit

### 4.1. Repair

Designed for structural applications, ECCs have unique properties suitable for applications in repair and retrofit of existing structures as well as for new structural applications. Here, we review highlights of studies in ECC repair and retrofit, emphasizing the translation of material ductility into structural system performance. Details of the ECC repair study can be found in Ref. [21]. Details of the retrofit study can be found in Refs. [22,23].

The most urgent need in concrete repair system is durability. It is generally recognized that the bond between the repair material and the substrate material is most important [24]. Failure can initiate from an interfacial defect causing delamination in the case of a weak ‘bond’ and spalling in the case of an overly strong ‘bond’ and a brittle repair material. At the moment several types of bond tests are recommended to obtain a ‘bond strength’ between the repair material and the substrate concrete.

Table 1  
Example of matrix mix proportion

W/C (%) (1)	Cement (2)	Silica fume (3)	Water (4)	Super plasticizer (5)
27 <sup>a</sup>	0.8	0.2	0.27	0.040

<sup>a</sup> Silica fume is included in cement weight.

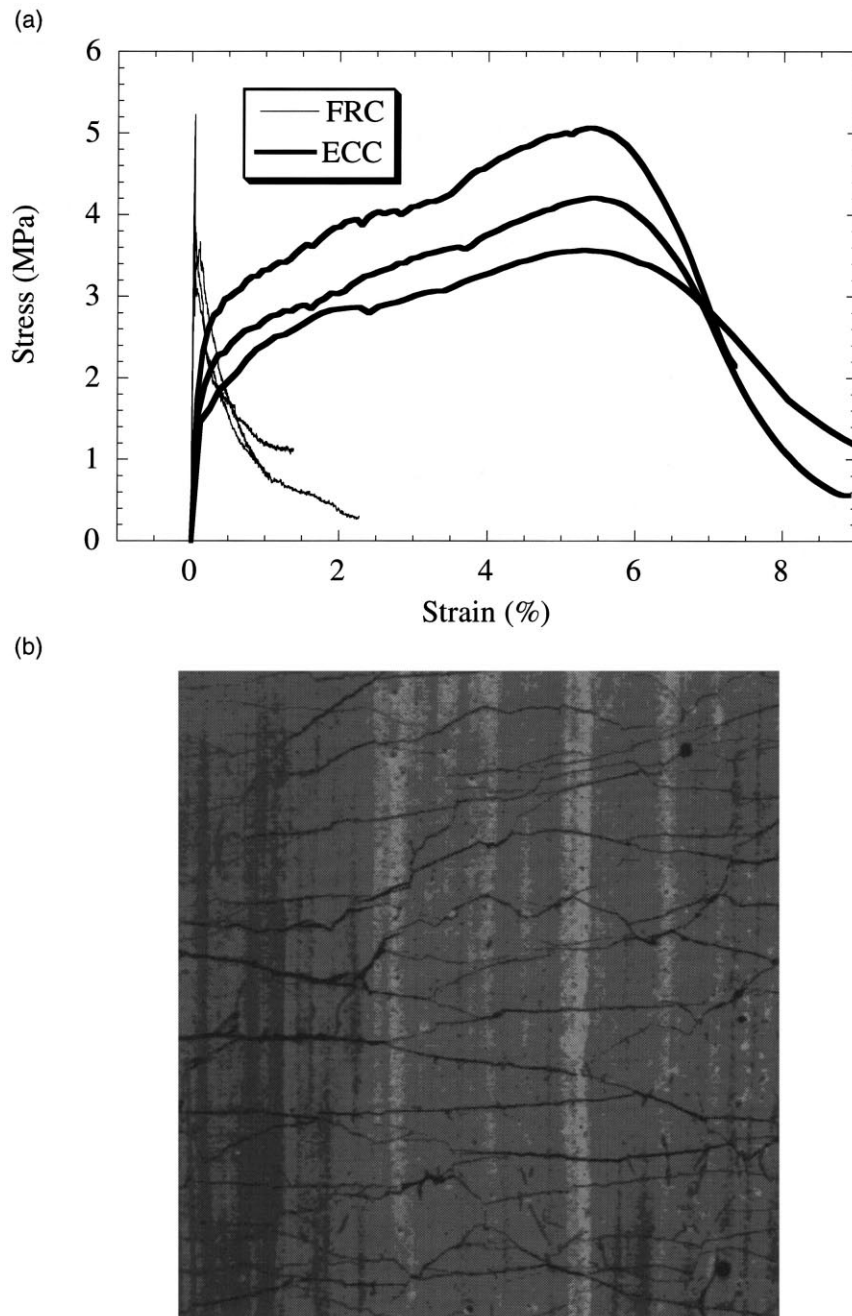


Fig. 3. (a) Uniaxial tensile behavior of ECC, and (b) microcracking with spacing of about 1 mm on tension specimen.

Unfortunately, there appears significant difficulty in transforming what appears to be a strong bond in the laboratory to durable repair performance in the field. There are two issues here. (1) Mechanics: If failure of the repaired system (delamination or spalling) is governed by a fracture process, characterization of the interface by a bond strength becomes questionable and strong size effect of the measured bond strength can be expected. This may in fact be the reason why bond strength test does not produce predictable results under field conditions, as the laboratory specimen size and geometry, loading configuration, or flaw size can be expected to be quite different from those in the field. (2) Materials: Since elimination of delamination naturally gives preference to spalling and vice versa, it becomes a dilemma that both material failure types cannot be eliminated simultaneously.

It is proposed here that ECC may offer the possibility in resolving the delamination/spalling dilemma. This is best explained by showing the test results of a simulated repaired system (see Fig. 6). Three repair material, plain concrete, FRC and ECC are used as overlays. To simulate an interface defect, a

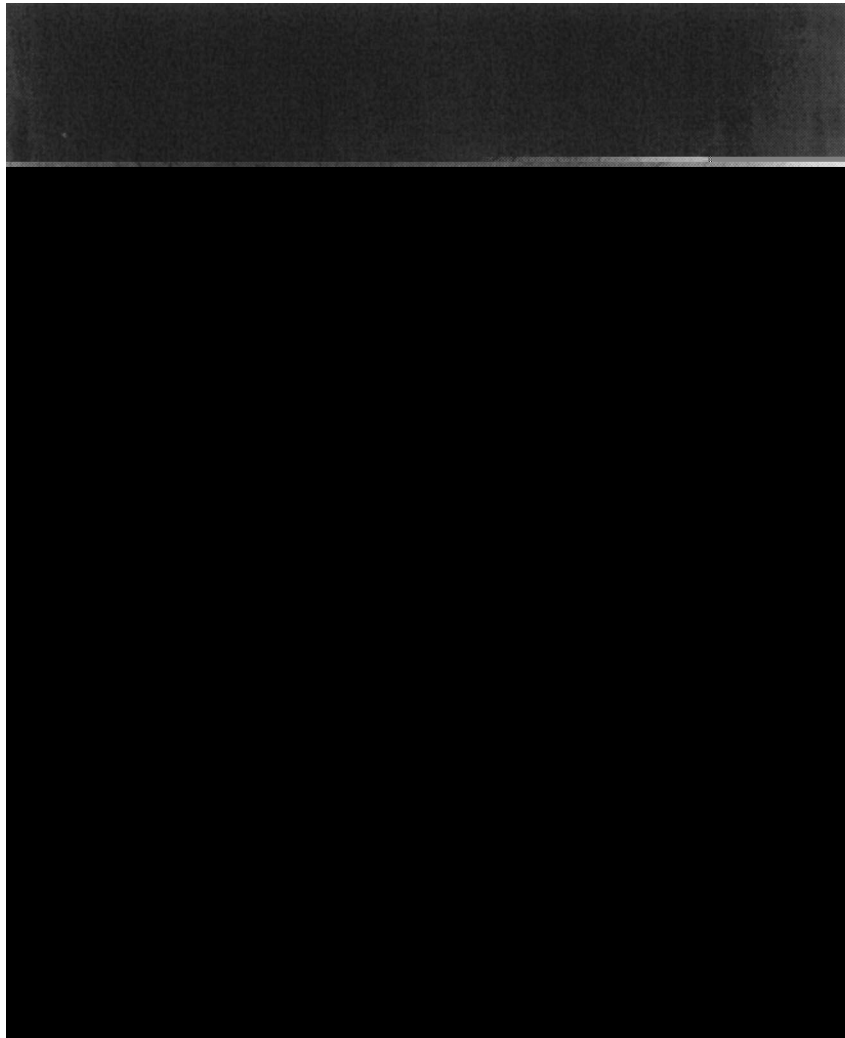


Fig. 4. Ductile fracture behavior of ECC. Scale marker is 50 mm.

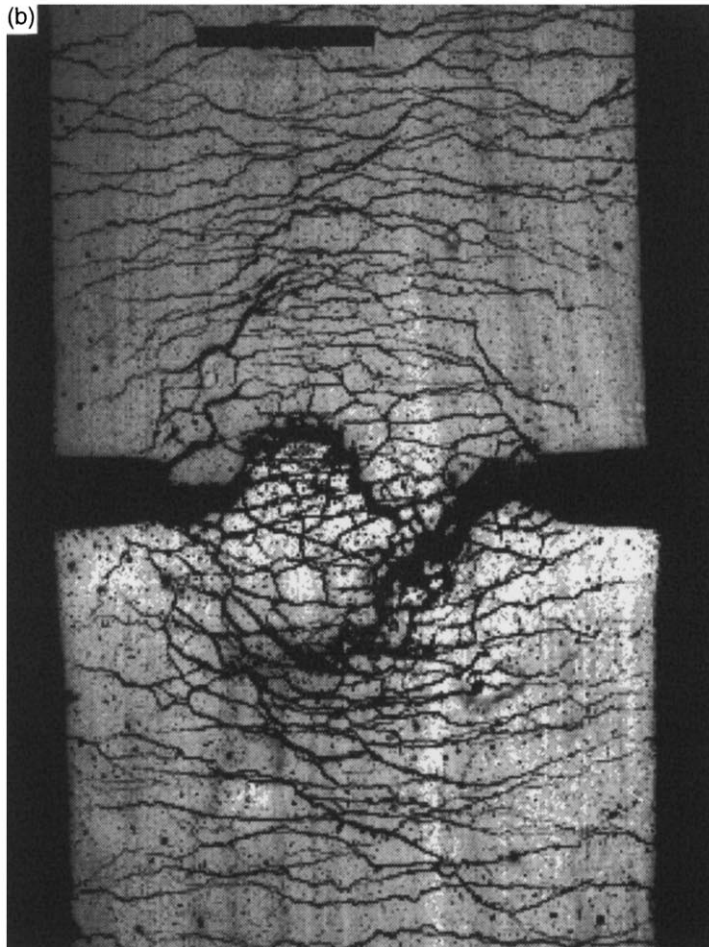
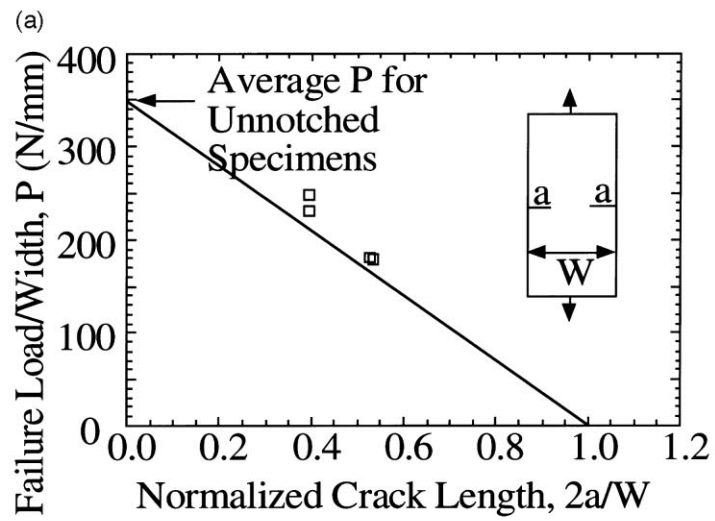


Fig. 5. Damage tolerance of ECC. (a) Tensile failure load as a function of ligament length; and (b) Specimen showing spread of damage away from notch plane. Specimen width is 75 mm. Marker is 20 mm.



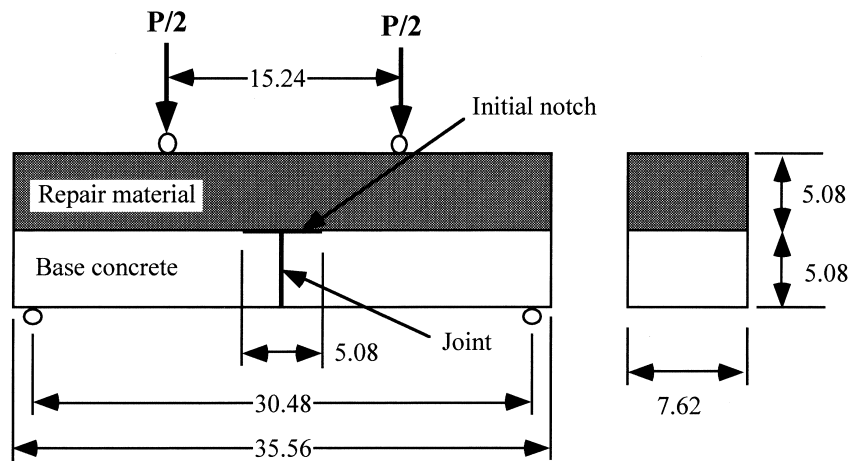
horizontal initial notch (using a tape with very smooth surface to prevent bonding between the base concrete and the overlay) is introduced at the interface. To deliberately represent a severe loading situation, a joint is included in the base concrete below the interface notch. A four point loading then introduces a mixed mode load on the interface notch. Fig. 7 shows the failure modes of the three repaired systems. For control specimens with concrete or FRC as the repair material, spalling of the repair material results, with very small amount of delamination. The high phase angle gives preference to kinking of the interfacial crack into the repair material [21]. For the concrete repair material, the load drop occurs immediately following the kink-out and the specimen broke into two halves. For the FRC repair material, the spall crack is bridged by the steel fibers and gradual load drop occurs as the spall propagates to the surface. For the ECC repair specimen, a sequence of interface crack extension, kink-out, kink-crack arrest, and re-initiation of interface crack events occur, under increasingly higher applied load. As a result, a pattern of kink-out microcracks in the ECC more or less following the interfacial crack tip is revealed. Fig. 8 shows a close-up view of the kink-out cracks. Small load drops appear to accompany the kink outs in the load–deflection curves measured in this system. Fig. 9 compares the very different load–deflection curve of all three sets of specimens.

One interpretation of this unique behavior of ECC as a repair material is based on the concepts of interface crack kinking suggested by [25]:

$$\frac{G}{G_{\max}^I} < \frac{\Gamma(\psi)}{\Gamma_c} \quad (1)$$

Where  $G$  is the energy release rate for driving the delamination interface crack,  $G_{\max}^I$  is the energy release rate (maximum at the most favorable angle) for driving the spall crack into the repair material,  $\Gamma(\psi)$  the interfacial fracture toughness and  $\Gamma_c$  is the toughness of the repair material.

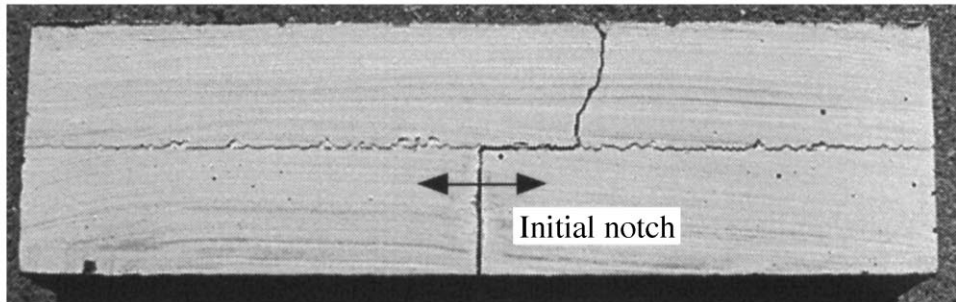
A plausible scenario is as follows: Loading increases linearly until  $G_{\max}^I = \Gamma_c$ , while simultaneously the kink condition ( Eq. (1)) is satisfied. This means that the interfacial defect finds it energetically preferable to kink into the ECC because of the low cement toughness (a low  $G_c$ ). However, as soon as the cement kink crack is formed, this ‘crack’ is bridged by fibers with bridging stress so strong that opening of the crack is accompanied by rising traction across the crack. (Straightly speaking, this is not



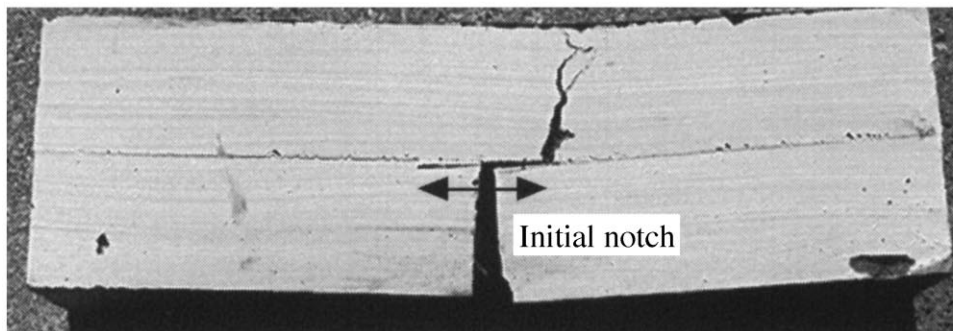
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Fig. 6. Simulated repair system.

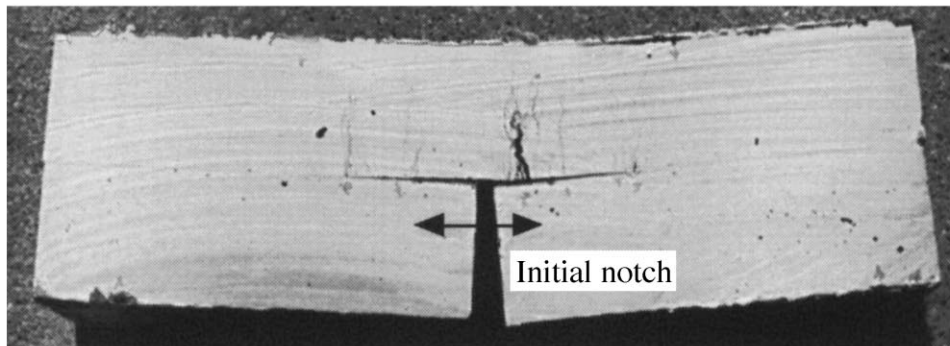
a real crack in the sense that traction is increasing with opening). Energetic consideration implies that Eq. (1) is no longer satisfied so that delamination is preferred, but only after a certain amount of load increase is imposed. As delamination reinitiates, once again the interfacial crack probes and finds it energetically preferable to kink into the ECC, once again because of the low cement toughness. This process can be repeated with load increase accompanied by small sudden drops whenever kinking



(a)



(b)



(c)

Fig. 7. Failure modes of (a) concrete/concrete; (b) FRC/concrete, and (c) ECC/concrete.

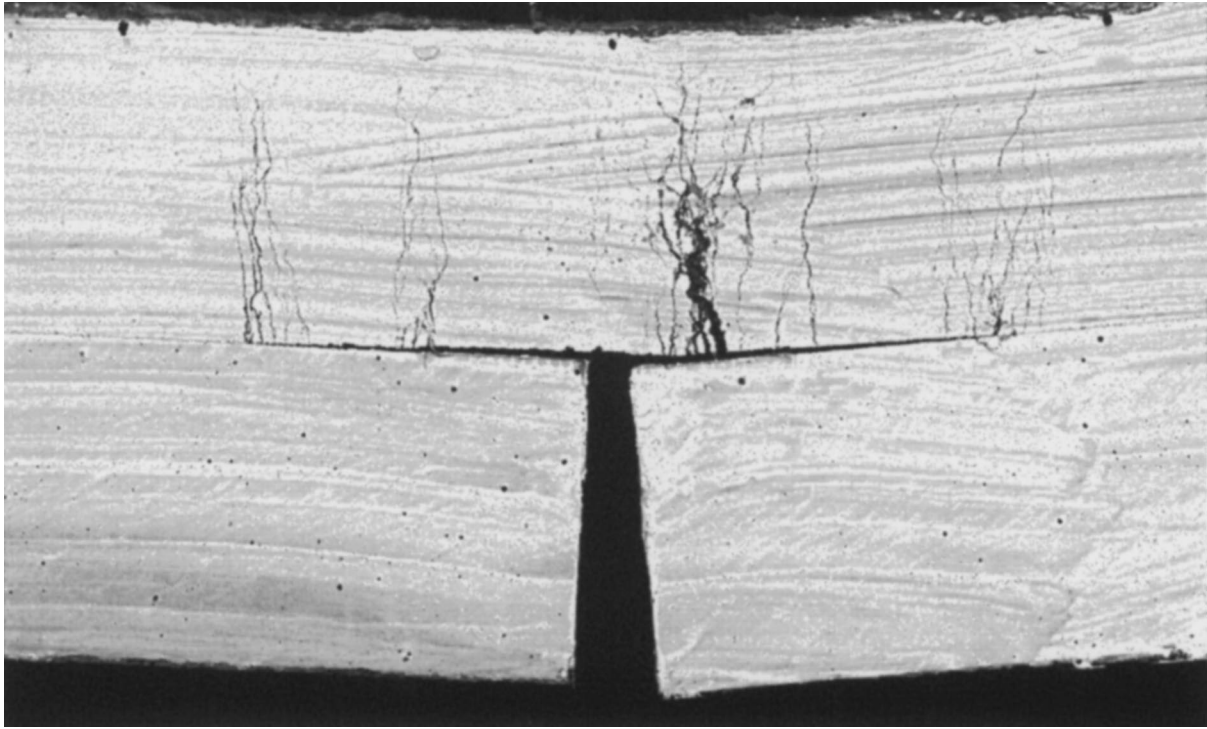


Fig. 8. Closeup of crack pattern in conc./ECC.

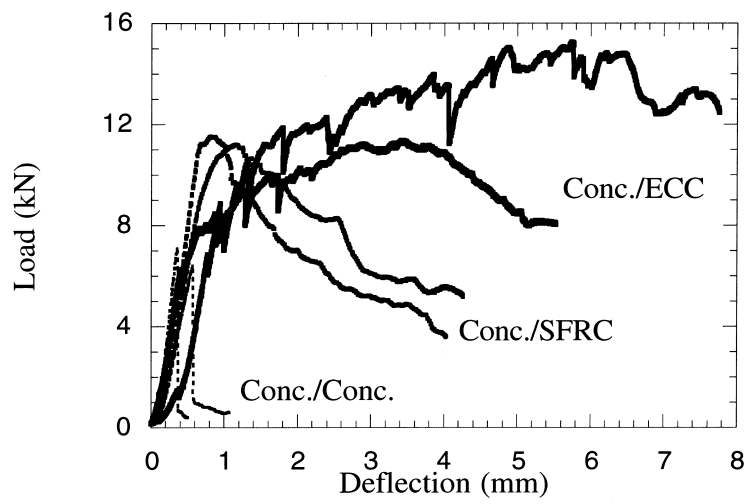


Fig. 9. Load–deflection curves of the three repaired systems.

occurs. On the specimen, we should expect a sequence of kink-out cement ‘cracks’ (Fig. 8). Final failure for the specimen shown is due to a flexural crack. This concept of kink crack sequence is corroborated with another type of bimaterial specimen used to determine the interfacial toughness of ECC/concrete. At high phase angle ( $\psi = 60^\circ$ ), when the kinking tendency is strong, a series of scale marks can be observed on the fracture surface by post-mortem examination (see Fig. 10).

We conclude therefore that the peculiar low initial toughness (that of the cement paste or mortar) and the strong bridging action of ECC together induce a kink-crack trapping phenomenon not present in the other cementitious repair materials tested. This kink-crack trapping mechanism may break the log-jam of the delamination/spalling dilemma, producing a very durable repair material which consumes significant amount of energy in the failure process. In the above discussion, we have depended on the concepts of interface fracture mechanics. If failure in repaired concrete system is governed by fracture, then the value of traditional bond test for bond ‘strength’ may be called into question. This area warrants further research in both the mechanics and material aspects. Such research can lead to significant enhancements to the durability of repaired pavements, bridge decks and other infrastructures.

#### 4.2. Retrofit

Many R/C buildings in the US and Japan have open beam-column frames which may be filled with non-structural partitioning walls. For building safety during seismic loading, the need to retrofit such buildings with shear structural walls has been recognized. Performance requirement includes shear wall integrity maintained up to 1 to 2% interstory drift, under full load reversals. To use the structural wall as an energy absorption device, it is critical to initiate inelastic deformation of the wall at shear strain much less than 1% (although not so low as to crack under normal service conditions). In addition, the wall must be designed so that this inelastic deformation takes place prior to damage to the beam-

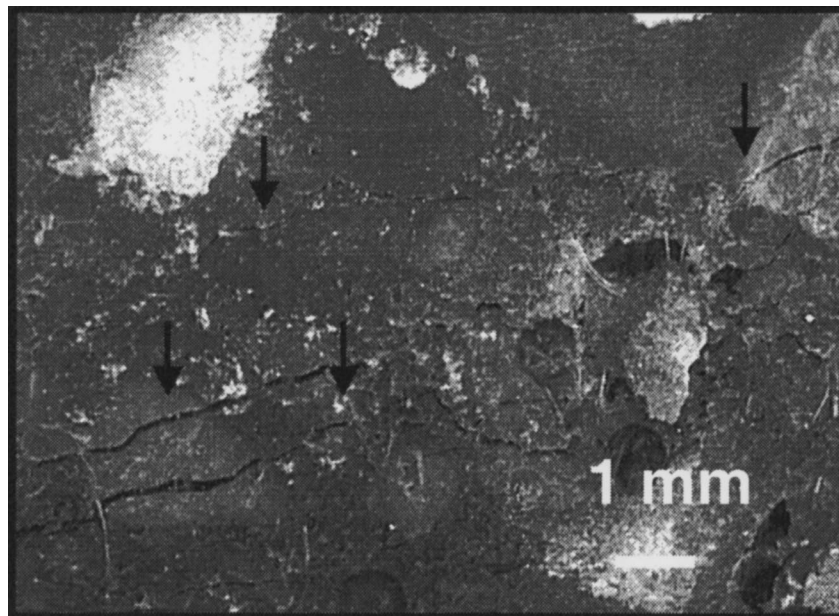


Fig. 10. Repeated kink crack scale marks left on specimen fracture surface.

column frame. For these reasons a lower first crack strength for the ECC is desirable (Fig. 2). Finally, for retrofitting an occupied building, it is necessary to install the shear wall rapidly.

One possibility being considered is to assemble pre-cast ECC panels on site [22]. Kabele et al. [26] studied the failure process of an ECC panel using FEM combined with a constitutive model developed especially for ECC material. In this model, the fixed smeared crack concept is used to represent multiple cracking of ECC. It is considered that cracks occur on planes perpendicular to the maximum principal stress direction, when its magnitude reaches the first crack strength. The overall stress–strain relationship in the crack normal direction is then hardening, as measured in a uniaxial tension test. Once the composite tensile strain capacity is exhausted, the normal stress starts to decrease, following tension–softening relationship, and fracture localization takes place. Reduction of the composite shear stiffness during both multiple and localized cracking phase is accounted for by assuming that shear stress is transferred across cracks only by bridging fibers when the cracks open. The ECC's nonlinear behavior in compression, including crushing, is modeled as plasticity.

The analysis of the pre-cast ECC panels was conducted under simplified loading and boundary conditions; shear was applied through a frame consisting of rigid elements connected by hinges, as shown in Fig. 11. For contrast, a quasi-brittle FRC panel under monotonic load was also studied [23]. Results of the analyses are summarized in Fig. 12 which shows the much greater (almost three times) shear load carrying capacity of the ECC despite similar (first crack and compressive) material strength to the FRC. The FRC panel failed by the joining of the shear cracks induced by the stress concentration at the ends of the construction joints. For the ECC panel, the stress concentration was relieved by the strain redistribution process of the ECC. Prior to failure, three diagonal bands of distributed tension microcracks had formed. At the peak load, compressive strain capacity had been exceeded in limited

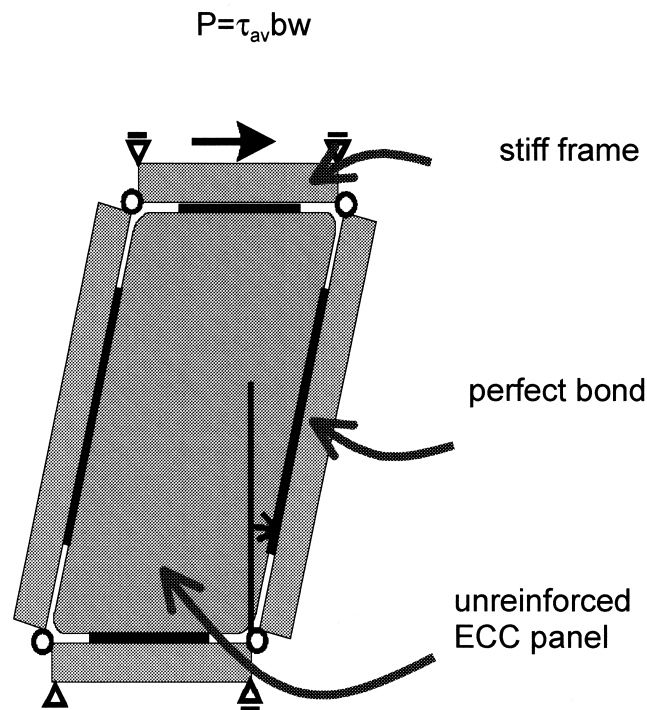


Fig. 11. Simplified model of shear panel.

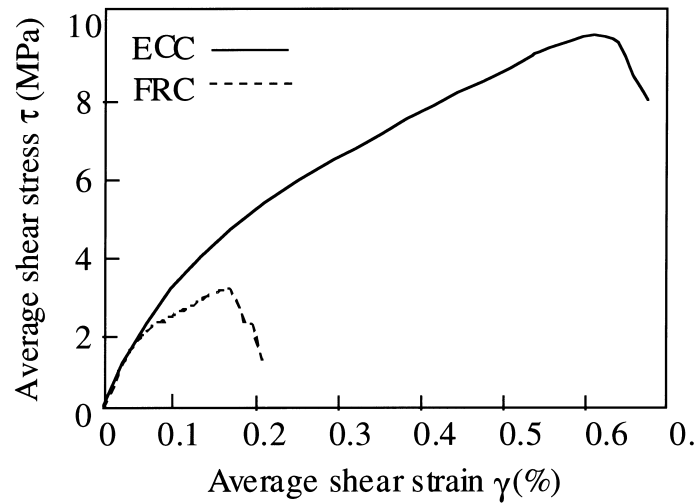


Fig. 12. Computed shear load capacity of wall panels.

areas near joints. At the same time, the maximum tensile strain near the joints reached the tensile capacity of 1.4%, suggesting initiation of fracture localization. Under reversed shear loading, the predicted energy dissipation in the hysteresis loop of the ECC panel is much higher than that of the FRC panel (see Fig. 13) [26].

Concrete elements generally are not suitable for dry jointing with high tension steel bolts. The unique strain-hardening and damage tolerant behavior of ECC shown in Fig. 5, however, may make tension-bolt with connection plates (see Fig. 14) suitable for joining ECC panels. To test this concept and to determine the maximum tension bolt force, indentation tests were conducted. The details of this test can

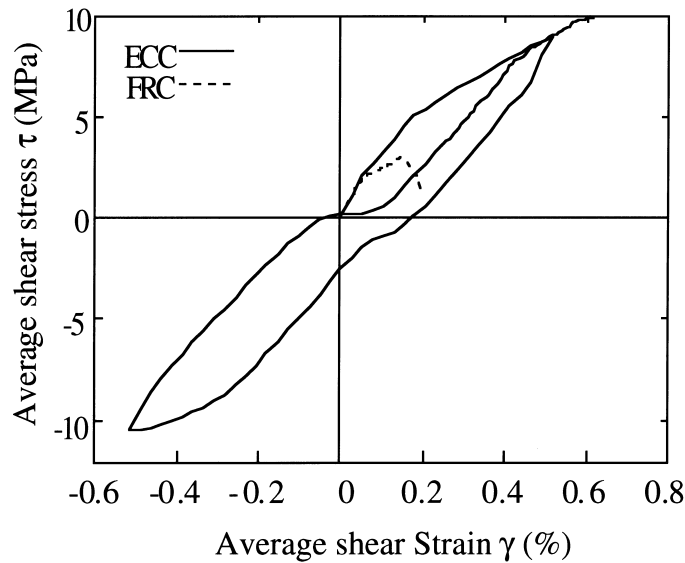


Fig. 13. Predicted hysteresis behavior of ECC panel.

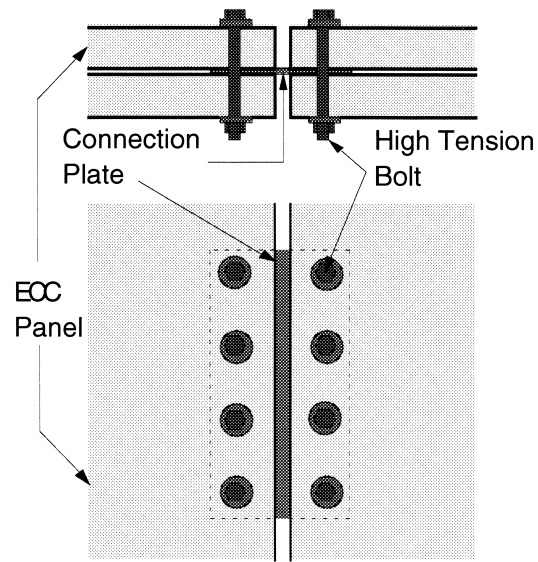


Fig. 14. Dry joint configuration.

be found in [26]. For an indent area of 1% of the slab, the failure load (see Fig. 15) for the ECC was twice (140 kN) that of the mortar (about 70 kN). The mortar has identical mix proportion of the ECC's matrix. Compressive strengths are 35 MPa for the ECC and 26 MPa for the mortar. Here, we note the very ductile failure mode of the ECC slab in comparison with the brittle fracture failure of a mortar control specimen (see Fig. 16).

In summary, ECC proves to be a suitable material for energy absorbing structural shear wall for

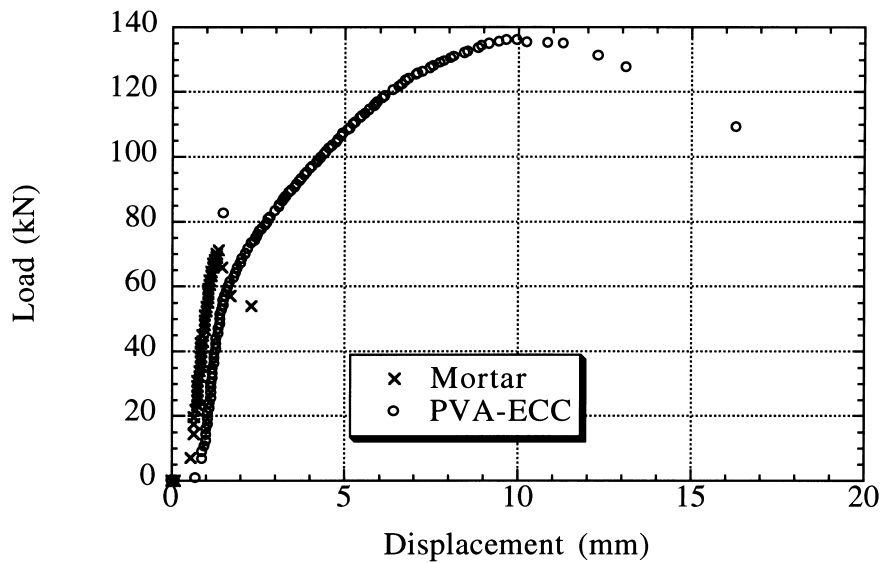


Fig. 15. Load–displacement curve of indent test.

seismic retrofit of open frame R/C buildings. The damage tolerant behavior of ECC delays fracture failure at the joints. A low first crack strength of ECC is actually advantageous in this application as inelastic deformation and energy absorption begin at moderately low building frame shear distortion. Also, the lower elastic modulus of the ECC in the strain-hardening stage lowers the possibility of damage on the frame due to the shear wall. Owing to the strain-hardening nature of ECC, wall integrity is expected even under full load reversals, as suggested by Fig. 13 and Ref. [26]. (Experimental information on the response of ECC under reverse cyclic shear load can be found in [22,27] and [28]). The analyses also suggest that conventional steel reinforcement may not be necessary for the shear panels. These results will need to be confirmed by experiments of prototyped walls. The experimental

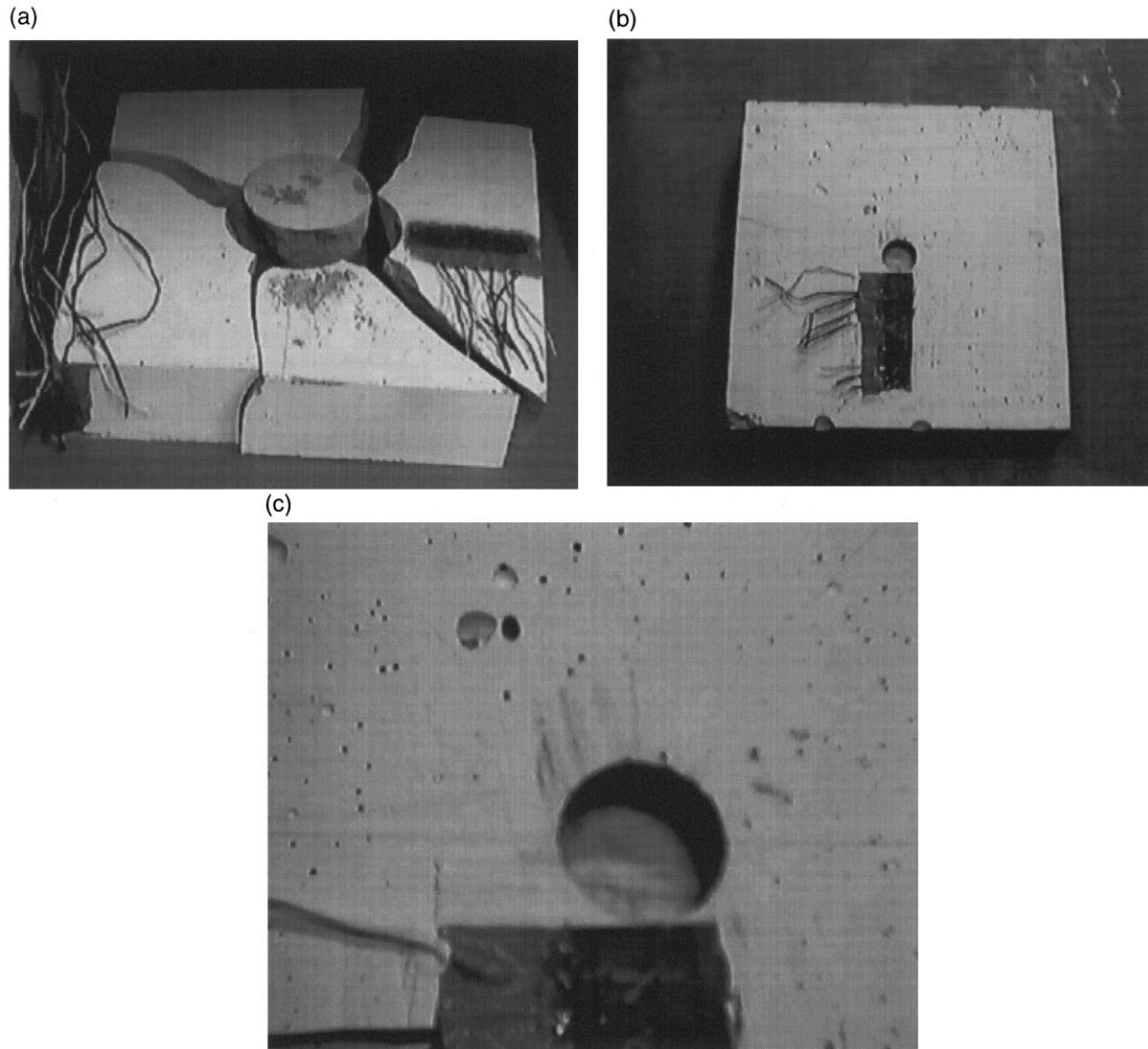


Fig. 16. Failed (a) mortar (b) PVA-ECC specimen (c) close-up of (b) near indenter.



indentation study confirms that dry jointing is applicable to ECC panels. Overall, therefore, the unique damage tolerant behavior of ECC makes it a suitable material for seismic retrofit applications.

## 5. Further observations and conclusions

Based on the above discussions, ECC has unique properties that can contribute to repair and retrofitting of structures. However, its applications are not limited to existing structures. New structures with performance requirements associated with large energy absorption, high impact resistance, large imposed deformation, crack width control, and large damage tolerance such as in hybrid (steel/concrete) structures can be potential targets of utilizing the unique properties of ECC.

Because of the strain-hardening property of ECC, this ductile materials behaves more like steel than traditional concrete. As a result, the reliability of the material is greatly enhanced. Hence design of ECC structures will also need to take into account these different features of ECC in order to optimally translate the high performance of ECC into high performance of ECC structures. This means that traditional design methodology used in concrete or R/C structures may need to be modified.

Perhaps equally important, ECC can be utilized together with other high performance material such as FRP. Very little has been explored in such combinations so far, but the opportunity of innovation is very real.

It should be pointed out that in both examples of repair and retrofit discussed in this article, the revelation that the structural strength is not governed by the material strength only should be clear. Although the ECC has tensile or compressive strength not very different from the FRC or mortar used in comparison cases, structural strength and structural ductility for the simulated repair overlay system (Fig. 9), the shear panels (Fig. 12), and the simulated bolt jointing system (Fig. 15), are all much higher when ECC is used. This drives home the point that material ductility is critical for high structural performance.

Finally, the enhancement of the performance of repaired or retrofitted structural systems reported here are not accidental. Instead, the use of micromechanics in tailoring the ECC composition (fiber, matrix and interface) is critical in achieving the tensile and damage tolerant properties. The examples of repair and retrofit illustrate the various scale linkages between structural performance, composite macro behavior and material micromechanisms.

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