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Composites Science and Technology 62 (2002) 775–781

COMPOSITES SCIENCE AND TECHNOLOGY

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Effect of inclination angle on fiber rupture load in fiber reinforced cementitious composites

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Received 7 August 2001; received in revised form 24 January 2002; accepted 20 February 2002

Abstract

A model has been formulated for analyzing the influence of fiber inclination angle on its rupture load in fiber reinforced cementitious composite. As a stiff fiber is pulled with an angle to the direction of pulling, the fiber rupture load decreases compared to the case with zero inclination angles. This phenomenon is called fiber apparent strength (defined as rupture load divided by fiber cross-section area) degradation. A parametric study, including the influence of elastic moduli of fiber and matrix, fiber/matrix interfacial bond strength as well as fiber orientation in matrix on the fiber apparent strength is carried out. The model results indicate that with the increase of fiber inclination angle, fiber apparent strength decreases. In addition, fiber apparent strength degradation degree is influenced by the elastic moduli of fiber and matrix and fiber/matrix interfacial bond strength. A fiber rupture test of carbon fiber in cementitious matrix was used for model verification. Model predictions based on independent parametric input compare favorably with experimental measurements of fiber apparent strength. © 2002 Elsevier Science Ltd. All rights reserved.

Keywords: A. Short fiber composites; B. Strength; Fiber apparent strength

1. Introduction

Incorporation of fibers in cement-based matrix has been found to improve several of its properties, primarily toughness, pseudo strain hardening, shear strength as well as impact and fatigue resistance [1–4]. For this reason fiber reinforced cementitious composites is now being used in increasing amounts in structures. The major effect of fibers is to act as bridging elements (ligaments) behind crack tips to resist crack propagation. Therefore, the structural performance of fiber reinforced composites are strongly controlled by the bridging behavior of the fibers which cross a matrix crack. The composite bridging stress by fibers crossing a crack is composed of the number of unbroken bridging fibers and the bridging force of such fibers for a given crack opening.

In a composite with randomly distributed fibers, the fiber inclination angle, i.e. the orientation angle relative to the normal of the crack plane, can have any value between 0 and $\pi/2$. Various investigations have shown that fiber inclined at an angle to the matrix crack plane leads to an increase in the bridging force. The mechanisms of the above results may include two parts. First, as described by Morton et al. [5], Brandt [6] and Li et al. [7], when a fiber is pulled out at an angle to the crack plane, there is an additional normal force acting on the surface of fiber which is provided by the matrix wedge near the fiber exit point due to the change of direction in fiber pull-out. Thus, a complementary friction is developed between fiber and matrix during fiber pullout. The influence of this friction on the bridging force depends on the interfacial properties of fiber and matrix and the inclination angle. According to Li et al. [7], this effect could be incorporated into the pullout force by writing

$$P(w, \phi) = P(w, \phi = 0)e^{f\phi} \quad (1)$$

Where f is associated with the local friction between fiber and matrix at the fiber exit point, the so-called snubbing coefficient that can be considered as one of the

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0266-3538/02/$ - see front matter
PII: S0266-3538(02)00045-3

fiber/matrix interaction parameters and which can be determined experimentally. Second, when non-aligned rigid fibers, such as carbon and glass fibers, are used, an additional force arises due to bending of the fibers as the fibers will have to bend at the cracked surface. The effect of fiber bending on bridging stress was studied by Morton and Groves [5]. In general, the bridging force provided by an inclined fiber can be considered as the vector sum of two components (see Fig. 1): a debonding component along the axis of the fiber and a bending component perpendicular to the fiber direction.

For an inclined flexible fiber, such as polyethylene (PE) and polyvinyl alcohol (PVA) fibers, the bending component becomes negligible compared to the debonding component, which is only applied by the local snubbing friction at the exit point of the fiber [8]. In this case, the fiber can be considered as a flexible string passing over a frictional pulley, and a snubbing friction model can be derived to relate the peak pullout load to the fiber inclination angle [7]. For stiff and brittle fibers, such as carbon and glass fibers, the bending component cannot be neglected, as the fiber does not bend completely over the pulley due to its high stiffness [9]. Based on above descriptions, it can be concluded that fiber inclination can lead to increase in bridging force, but it can also lead to higher stress on the fiber due to bending and therefore higher fiber rupture potential, thus reducing the number of unbroken fibers.

A number of studies had been made in the past to evaluate the effect of fiber inclination angle on crack bridging stress in fiber reinforced composites. However, the effect of fiber inclination angle on fiber rupture has so far received very little attention. When the fiber axis is inclined at an angle to the crack plane, fiber flexural tensile failure or flexural compressive failures may occur, which lead to the fiber rupture load lower than that of the aligned fiber, so called apparent strength degradation [10]. The fiber apparent strength reduction certainly will influence the fiber bridging stress because this governs the amount of fibers being pulled out. Therefore, this has to be taken into account in composite design. Fiber apparent strength reduction had been found in glass-polyester, carbon–cement composites due to the bending moment [9,11]. To properly determine the fiber bridging stress on crack plane, it is necessary to account for any fiber strength reduction, since the fiber bridging stress is related to fiber strength, embedded length, bond strength and fiber inclination angle to the crack plane.

In the present paper, the analytic model based on the cantilevered beam representation of the fiber will be adapted to analyze the rupture load of the stiff fiber, such as carbon and glass fibers under different inclined angle. The principle goal of the present work is to clarify the bending resulted fiber apparent strength reduction, including influence of fiber and matrix as well as fiber/matrix interface properties, such as elastic modulus of fiber and matrix, fiber/matrix bond strength. In addition, the influence of fiber inclination angle on fiber critical embedment length is discussed.

2. Derivation of the model

In the analysis, the fiber is separated into two free bodies at the middle of the crack (Fig. 1). Similar to the approach proposed by Katz and Li [9], each part of the fiber is then simulated as an elastic beam, partially supported on a matrix foundation and partially cantilevered free. Following Morton and Groves [5], the deflection at free end, δ, and free portion length, l, can be expressed in terms of the crack width, w, and the fiber diameter, d_f, as:

$$\delta = \frac{1}{2} w \sin(\theta), l = \frac{1}{2}\left[d_f \tan(\theta) + w \cos(\theta)\right] \tag{2}$$

With reference to Fig. 2, the cantilevered part of the beam is subjected to axial load, N and bending load P. The supported part is subjected to the axial load N, shear load V, and bending moment M, transferred from the cantilevered part.

The deflection of the supported part as a result of shear load V and bending moment M are [12]

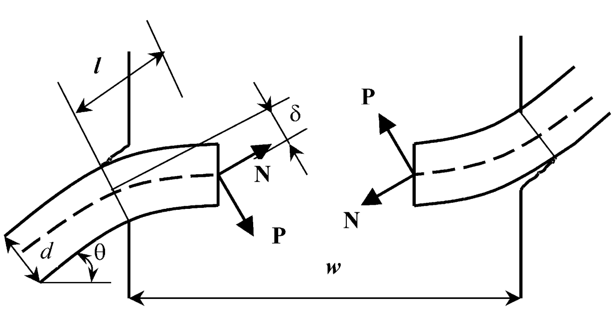

Fig. 1. Bending of fiber across crack.

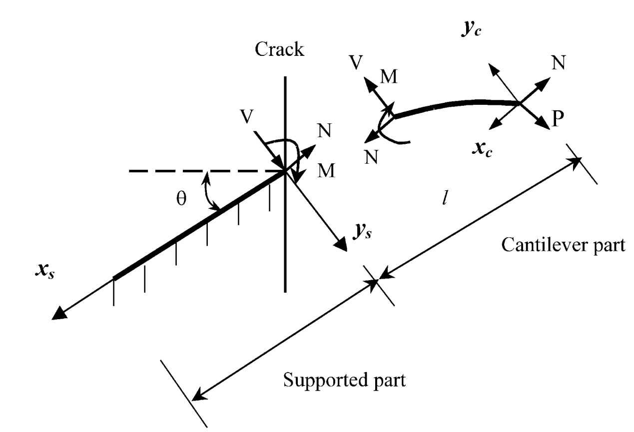

Fig. 2. Loads of supported and cantilevered parts of fiber as pulled.

$$y_s = \frac{2\lambda}{k}\left[(V+\lambda M)e^{-\lambda x_s}\cos(\lambda x_s) - M\lambda\sin(\lambda x_s)\right] \tag{3}$$

where $\lambda = \sqrt[4]{k/4E_fI_f}$; k is stiffness of matrix foundation; and E_f and I_f are the modulus of elasticity and moment of inertia of the fiber, respectively.

For the cantilevered part of the fiber, the moment in the fiber can be expressed in terms of bending load P and axial load N at the fiber end, $x_c = 0$, i.e.

$$M(x_c) = -E_fI_f\frac{d^2y_c}{dx_c} = -Ny_c + Px_c \tag{4}$$

Solving Eq. (4) with boundary conditions of $y_c = 0$ at $x_c = 0$, the deflection along the cantilevered part can be expressed as

$$y_c = 2C\sinh(mx_c) + \frac{P}{N}x_c \tag{5}$$

where $m = \sqrt{N/E_fI_f}$.

From Eqs. (3) and (5), using the conditions

$$\frac{dy_s}{dx_s}(x_s = 0) = \frac{dy_c}{dx_c}(x_c = l)$$
$$y_s(x_s = 0) + y_c(x_c = l) = \delta$$

and note $V = P$, the unknown moment M, bending load, P and integration constant C can be expressed as a function of axial load, N and fiber deflection δ as well as cantilevered length l. Finally, we get:

$$P = \frac{\delta}{K_2};\ M = -2K_1PN\sinh(ml) \tag{6}$$

where

$$K_2 = -\frac{4\lambda}{k}K_1N[m\cosh(ml) + \lambda\sinh(ml)] + 2K_1\sinh(ml) + \frac{l}{N}$$
$$K_1 = \left\{-\frac{4\lambda^2N^2}{k}[m\cosh(ml) + \lambda\sinh(ml)] - 2Nm\cosh(ml)\right\}^{-1}$$

Here δ and l can be related to crack opening, w and inclined angle θ by Eq. (2). The axial load N can be related to crack opening w for a given fiber-matrix bond strength τ and chemical bond energy G_d [13], that is

$$N = \frac{\pi}{2}\left[(1+\eta)E_fd_f^3\left(\tau w + \frac{2G_d}{1+\eta}\right)\right]^{\frac{1}{2}}e^{f\theta} \quad \text{for } 0 \leqslant w \leqslant w_{fd}$$
$$N = \pi d_f\tau(L - w + w_{fd})e^{f\theta} \quad \text{for } w_{fd} < w < L + w_{fd} \tag{7}$$

where w_{fd} is the crack opening corresponding to full debonding of shorter embedment length L of the fiber. It is given by

$$w_{fd} = \frac{4\tau L^2(1+\eta)}{E_fd_f} + \left[\frac{32G_dL^2(1+\eta)}{E_fd_f}\right]^{\frac{1}{2}} \tag{8}$$

$\eta = V_f\ E_f/V_m\ E_m$, V_f, E_f and V_m, E_m are the volume fractions and elastic modulus of fiber and matrix respectively. It should be noted that for some kinds of fiber/matrix systems G_d is equal to zero, such as carbon/cement matrix and steel fibers/cement matrix interfaces. And for some others, G_d is greater than zero, such as glass and polyvinyl alcohol fibers cement matrix systems. From (7), it can be seen that fiber bridging stress achieves maximum value at fiber full debonding point for most fibers with slip-softening fiber/matrix interface, for example glass, carbon and steel fibers in cementitious matrix. Next, we assume the maximum tensile stress due to axial pullout load and bending load occurs at fiber exit point. It is acceptable because even the maximum moment along the matrix foundation is located at a short distance in the matrix, but the axial load is reduced along this distance due to bond, which in turn compensates for the increase in the bending stress. The difference between real maximum stress and the stress calculated at the fiber exit point is less than 5% [9]. This difference is ignored without loss of accuracy. Then the maximum tensile stress in the fiber can be calculated by

$$\sigma_m = \frac{Md_f}{2I_f} + \frac{4N}{\pi d_f^2} \tag{9}$$

Thus, if $\sigma_m = \sigma_f$, the fiber will breakage. The corresponded maximum fiber bridging force F_m can be calculated through

$$F_m = P_m\sin\theta + N_m\cos\theta \tag{10}$$

Where P_m and N_m are the bending and axial loads corresponding to that σ_m equals to the fiber strength σ_f. If we further define the fiber apparent strength [10] σ_{fa} as F_m/A_f, then

$$\sigma_{fa} = \frac{1}{A_f}(P_m\sin\theta + N_m\cos\theta) \tag{11}$$

In addition, in terms of debonding length l, the axial load N can be expressed as [13]

$$N = \left[\pi d_f\tau l + \left(\frac{1}{2}\pi^2G_dE_fd_f\right)^{1/2}\right]e^{f\theta} \tag{12}$$

As N achieves N_m, the critical fiber embedment length l_c, which defines as the maximum shorter embedment length in matrix as fiber is pulled out instead of rupture, can be calculated by

$$l_c = N_m e^{-f\theta} - \frac{\pi}{\sqrt{2}}\left(E_f d_f G_d\right)^{1/2} / \pi d_f \tau \quad (13)$$

The determination of fiber apparent strength in matrix as a function of fiber inclination angle is performed according to the following algorithm. For given fiber inclination angle, sufficient fiber embedment length and fiber, matrix as well as fiber/matrix interface properties, the axial load N can easily be calculated by (7) with a given crack opening w $(0<w<w_{fd})$. The corresponded values, such as P, M and σ_m as well as F can then be determined by using (6), (8) and (10). With the increase of crack opening w, σ_m increases as well until the fiber strength σ_f is achieved. Thus, the fiber apparent strength can be obtained by (11). Corresponded critical fiber embedded length l_c can then be calculated by (13).

3. Model results and discussions

As an example, carbon fiber in cement matrix is used to analyze the influence of fiber orientation angle on its rupture load during fiber pullout. The fiber, matrix and fiber/matrix interfacial properties used in the analysis are listed in Table 1, where $k/E_m = 0.25$. Typical model result of inclination angle effect on carbon fiber rupture load in cement matrix in terms of fiber apparent strength and inclination angle diagram is shown in Fig. 3. The influence of inclination angle on fiber critical embedment length is shown in Fig. 4, where three different cases, i.e. with snubbing and bending effect, with snubbing effect only and without either of both are shown together. From the model result, it can be seen that the fiber apparent strength decreases with the increase of fiber inclination angle. The behavior of this fiber apparent strength degradation due to bending can be generalized as a fast dropping stage (within 10 °) followed by a relative stable decreasing stage. Using the parameters listed in Table 1 as model input, the fiber apparent strength is shown to reduce by 50% or more for fiber inclination angle greater than 10 °. The corresponded fiber critical embedded length changes from 26 to 10 mm as fiber inclination angle varies from 0 to 10 °. The fiber critical embedded length achieves the minimum value (5 mm) at 60–80 °. The result indicates that with the increase of fiber inclination angle (from 0 to 80 °), the maximum normal force N_m to break the fiber decreases. It is noted that as the angle greater than 80 °, the fiber critical embedded length starts to increase again with the increase of inclination angle. This physically indicates that the required normal force N_m increases with the increase of angle after 80 °. From the model predictions, it can easily find that under the same angle, the critical fiber embedment length in the threes cases are obviously different. Snubbing leads to critical fiber embedment length decrease and bending the stiff fiber leads to further reduction. The influence of fiber bending during pullout from matrix is more significant than the influence of snubbing for those kinds of fibers. These analytical results indicate that the influence of fiber inclination angle on both fiber rupture load and critical embedment length cannot be ignored. As fibers are pulled out from matrix, the higher the fiber orientation angle, the more the fibers are broken. This behavior has to be taken into account in material design with crack bridging mechanics of fiber reinforced composites. In the following, the influences of material parameters of both fiber and matrix as well as fiber/matrix interface on the fiber apparent strength are investigated separately.

Table 1
Material parameters used in the analysis

Items	d_f (mm)	E_f (GPa)	E_m (GPa)	k (GPa)	σ_f (MPa)	τ (MPa)	G_d (J/m^2)	f
Value	0.046	175	30	7.5	930	0.4	0	0.5

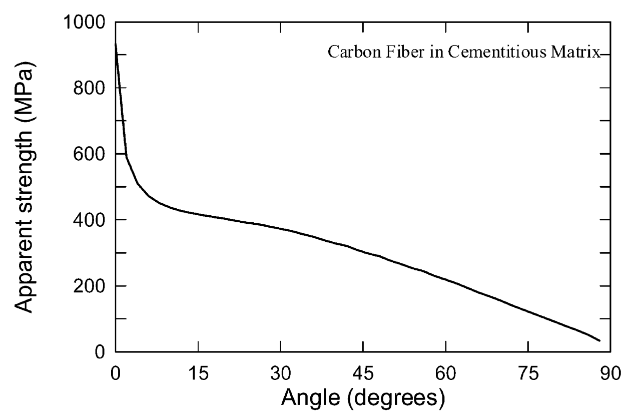

Fig. 3. Effect of inclination angle on carbon fiber apparent strength.

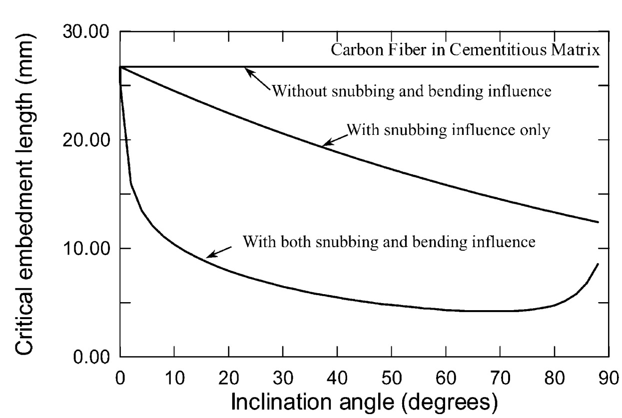

Fig. 4. Influence of inclination angle on carbon fiber critical embedment length.

3.1. Influence of elastic moduli of fiber and matrix

It is well known that the elastic modulus of cementitious matrix depends on a number of material parameters including water/cement ratio, sand content and matrix age since casting. Therefore the effect of matrix modulus on the fiber apparent strength degradation is obviously of importance. In addition, fiber elastic modulus is one of the principle material constituent properties. To study the effect of fiber modulus on fiber rupture load is valuable to both fiber development and composite performance optimization. The effect of matrix modulus on fiber apparent strength is shown in Fig. 5. The fiber parameters used in the calculation are the same as those given in Table 1 except for the change in the matrix modulus as indicated in the figure. From this model simulation, it can be seen that the fiber apparent tensile strength degradation is exasperated by the increase of matrix elastic modulus. It indicates that lowering the matrix modulus can lead to enhancement of the fiber apparent strength. Thus, the amount of fibers be pulled out instead of broken can be increased and the fracture toughness of the composite can then be improved. Therefore, it can be concluded that fiber behavior in mortar should be better than in concrete not only due to the difference on fiber volume content, but also due to the above reason. In addition, Observation of the influence of matrix modulus on fiber rupture may also be generalized to the increase potential of fiber rupture with age of composite, since the matrix stiffness should gain with time during curing.

Similarly, the influence of fiber modulus on apparent strength is shown in Fig. 6. The fiber and matrix parameters used in the modeling are the same as those listed in Table 1 except that the fiber modulus is treated as the variable parameter. From Fig. 6, we can see that with increase of fiber modulus, the fiber apparent strength increases. In the view of this point, rising fiber modulus is of benefit to improve composite performance.

3.2. Influence of fiber/matrix interfacial parameters

As an important micro material property, fiber/matrix bond parameter has been extensively investigated by researchers. Up to now, it is possible to tailor the fiber/matrix interface by a number of techniques, such as plasma treatment and fiber surface coating [14,15]. In this section, the influence of fiber/matrix interfacial bond strength on the fiber apparent strength degradation behavior will be studied using the above model. As an example, assuming a carbon fiber in cement matrix, the influence of fiber/matrix interfacial bond strength τ is shown in Fig. 7. The fiber and matrix material parameters used in the analysis are the same as those listed in Table 1 except that the bond strength is treated as a variable parameter. From Fig. 7, it can be seen that the influence of fiber/matrix interfacial bond strength on the fiber apparent strength and inclination angle relationship is quite significant. With the increase of bond strength, fiber apparent strength increases. Therefore, it can be concluded that increasing the fiber/matrix bond strength leads to a decrease in fiber apparent strength degradation due to bending. However, high bond between fiber and surrounded matrix may result in more fiber breakage than low bond case as fiber pullout from matrix. All material parameters have to be considered synthetically in material design.

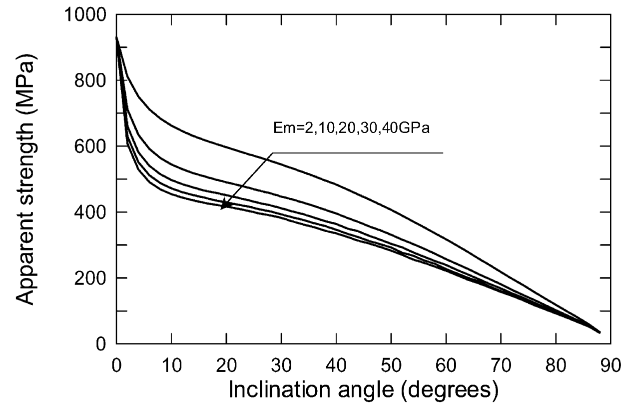

Fig. 5. Effect of matrix elastic modulus on fiber apparent strength.

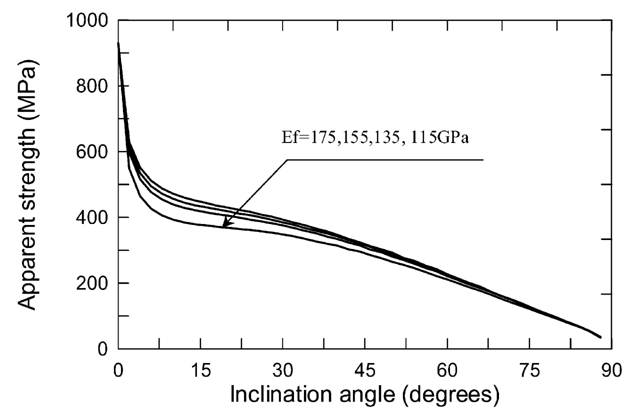

Fig. 6. Effect of fiber elastic modulus on fiber apparent strength.

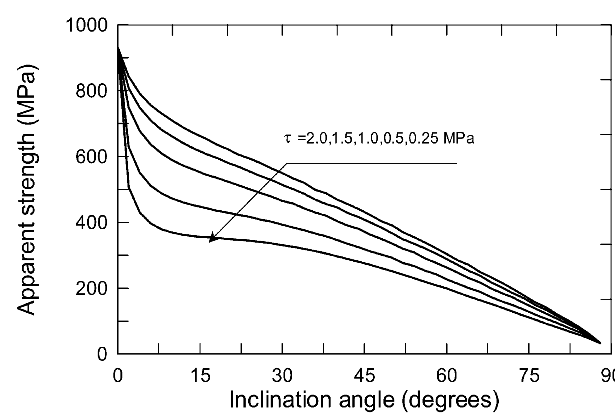

Fig. 7. Effect of fiber/matrix bond strength on fiber apparent strength.

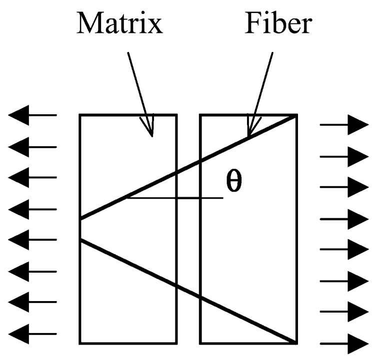

Fig. 8. Test setup for fiber rupture.

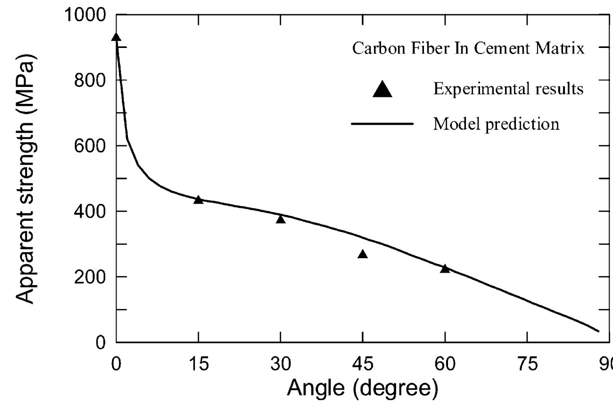

Fig. 9. Comparison between model prediction and experimental results on fiber apparent strength.

Table 2
Summary of material parameters used in the experiments

Items	d_f (mm)	E_f (GPa)	E_m (GPa)	k (GPa)	σ_f (MPa)	τ (MPa)	G_d (J/m^2)	f
Value	0.046	175	20	5.0	930	0.4	0	0.5

4. Experimental verification

Katz and Li [9] carried out an experimental investigation of the inclination angle effect on fiber rupture load. The experimental results obtained by the above study are compared with the model predictions. The experimental details are briefly summarized as follows.

Carbon fiber and cement matrix were used in their study. In the specimen, two symmetrical fibers were cast in matrix with a prescribed angle φ to the loading direction, see Fig. 8. Each part of the matrix was cast separately on subsequent day and a thin oil layer was applied between the two matrix parts to minimize the bond between them. The selected inclination angle was 15, 30, 45, 60 and 75 °. The specimen was glued to the test machine and direct tension was applied. The fiber tensile strength, elastic modulus and diameter was 930 MPa, 175 GPa and 0.046 mm, respectively. Type III cement past with 0.5 water/cement ratio was utilized to form the matrix. All specimens were kept in a humid environment until testing day. The test was carried at seven days after the second cast. More detailed description on the experiments can be found in [9].

The relevant material parameters of the above fiber reinforced concrete and mortar used in the model calculations are summarized in Table 2. It is noted that bond strength between fiber and matrix is based on the experimental results of single fiber pullout tests [16]. Because the matrix used in the experiments was cement paste without any aggregate, 20 GPa was used as the model input value of the matrix elastic modulus. Fig. 9 shows the comparison between model predictions and experimental results in terms of fiber apparent strength and inclination angle diagram. It can be seen that a good agreement between model predictions and independent test results is obtained. The present model can well predict the fiber apparent strength degradation behavior as fiber pullout from matrix with an inclination angle to the pulling direction.

5. Conclusions

An analytical study on the effect of fiber inclination angle in matrix on fiber rupture load has been presented. As an inclined stiff fiber is pulled, due to the effect of bending, fiber rupture load is significantly reduced compared to the case of fibers with zero inclination angle. This is called fiber apparent strength degradation. Due to the fiber apparent strength degradation with the increase of fiber orientation angle, the critical fiber embedement length decreases.

Parametric study with the present model yields the following conclusions.

With the same fiber geometry and inclination angle, the higher the modulus of matrix, the lower the fiber apparent strength. Conversely, with the increase of fiber modulus, the fiber apparent strength increases. An increase in the fiber/matrix bond strength leads to a higher fiber apparent strength.

Model predictions were compared with independent experimental results and good correlation between model and experiments was found. The model can be used in both material optimizing and performance prediction with regard to the influence of fiber inclination angle on fiber rupture load.

Acknowledgements

Support from the National Science Foundation (CMS-9872357) to the University of Michigan and from Scientific Foundation for Returned Overseas Chinese Scholars, Ministry of Education are gratefully acknowledged.

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